

# ProVerif と Tamarin-prover の観測等価性の違い

三重野 武彦 岡崎 裕之 布田 裕一 荒井 研一



# Contents

1. 発表の概要
2. 観測等価性とは
3. DDH仮定
4. Bi-systems
5. 検証概要
6. 検証の詳細結果
7. 結果の考察
8. まとめと今後の課題

# 1. 発表の概要

自由度の高い、  
形式的安全性検証を実現する



安全性検証に使用するツールの  
優位性や使い分けなどを探っている



トレース特性や観測等価性  
どのような違いがあるか

本講演では, **DDH** を使用して検証

## Tamarin-prover の概要



A multiset rewriting rule is described as follows.

- 1 rule Name:
- 2  $[Pre(x)] - [Action(x)] \rightarrow [Post(x)]$

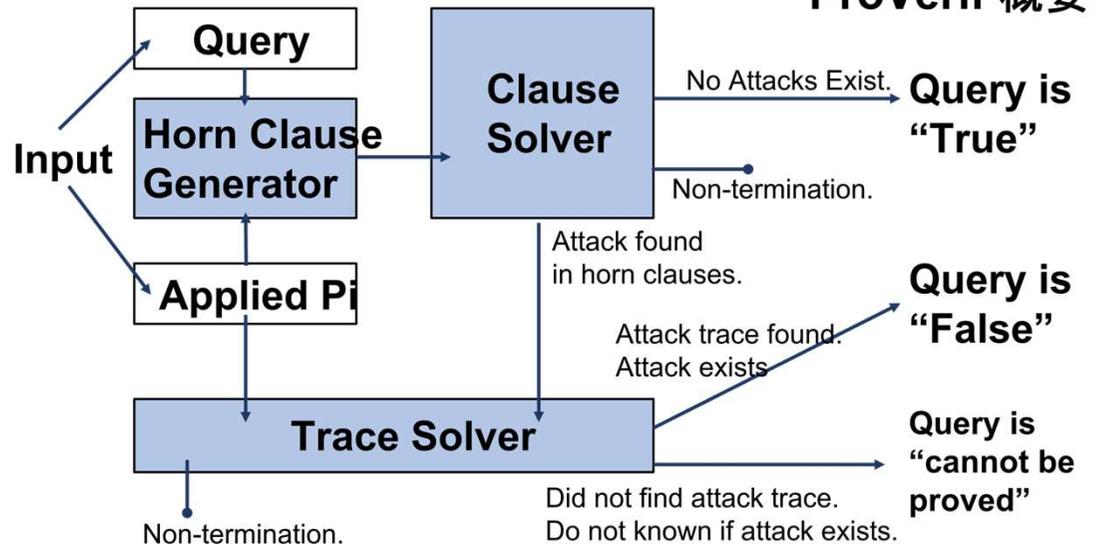
Lemma Annotations

lemma Name [Annotation1, Annotation2]:  
e.g.) lemma nonce\_secret:  
"All m #i #j. Secret(m) @i & Key(m) @j ==> F"

### Tamarin-prover query handling

$Secret(session1, key)@i \& Key(key)@j.$

### ProVerif 概要



Refer: Verification of security protocols with state in ProVerif : Avoiding false attacks when verifying freshness.

## 2. 観測等価性とは

プロセス計算に確率的な側面を導入し、プロセス計算の中で、計算論的な識別不可能性を観測等価性として表現した。

形式モデル:

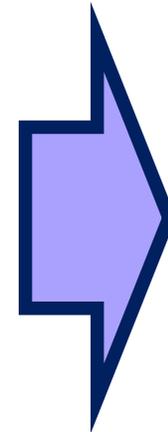
識別不可能性の概念: 一般的に観測等価性と表現 [参考文献.A]

プロセスAとBを区別出来ない

**A**  
✗



**B**  
✗



**A**  $\approx$  **B**

[参考文献.A]

JOHN C. MITCHELL, AJITH RAMANATHAN, ANDRE SCEDROV, AND VANESSA TEAGUE.  
"A PROBABILISTIC POLYNOMIAL-TIME PROCESS CALCULUS FOR THE ANALYSIS OF CRYPTOGRAPHIC PROTOCOLS",  
Theoretical Computer Science, Volume 353, Issues 1-3, 14 March 2006, Pages 118-164.

# 関連研究

## **ProVerif**

### 1. Proving More Observational Equivalences with ProVerif

Vincent Cheval and Bruno Blanchet,  
In Proceedings of the 2nd International Conference on Principles of Security and Trust (POST'13).

## **Tamarin-prover**

### 2. Automated Symbolic Proofs of Observational Equivalence

D.A. Basin, J. Dreier, and R.Sasse,  
In 22nd Conference on Computer and Communications Security (CCS'15),  
pages 1144–1155. ACM Press, 2015.

**比較には触れていない**

## **Other**

ProVerifを使用した観測等価性

・投票プロトコル自動検証 (Kremer, Ryan '05) , etc

### 3. DDH(Diffie-Hellman) 仮定

生成器 $g$ を持つ素数 $q$ の巡回群 $G$ が与えられたとき,  
 $(g^a, g^b, g^{ab})$ は $(g^a, g^b, g^c)$ と計算上区別がつかない。  
ここで  $a, b, c$  は  $Z_q^*$ からのランダム要素である。

DDH問題(判定DH)

$a, b, c \in Z$  をランダムに選択

$(g, g^a, g^b, g^{ab})$  と  $(g, g^a, g^b, g^c)$

を識別する問題



$(p, g, g^a, g^b, g^c, Z)$

DDH問題を解く  
アルゴリズム



0 or 1

# 検証結果

観測等価性は既に証明されている

## Tamarin-prover

```
=====
summary of summaries:
analyzed: fajs-tamarin-ddb.spthy
DiffLemma: Observational_equivalence : verified (2522 steps)
=====
```

## ProVerif

```
-----
Verification summary:
Observational equivalence is true.
-----
```

何故この結果？

# Biprocess 0 (the initial process)

{1} new a: exponent;

{2} new b: exponent;

{3} new c: exponent;

{4} out(d, (exp(g,a),exp(g,b),choice[exp(exp(g,a),b),exp(g,c)]))

## 4. Bi-systems の概要

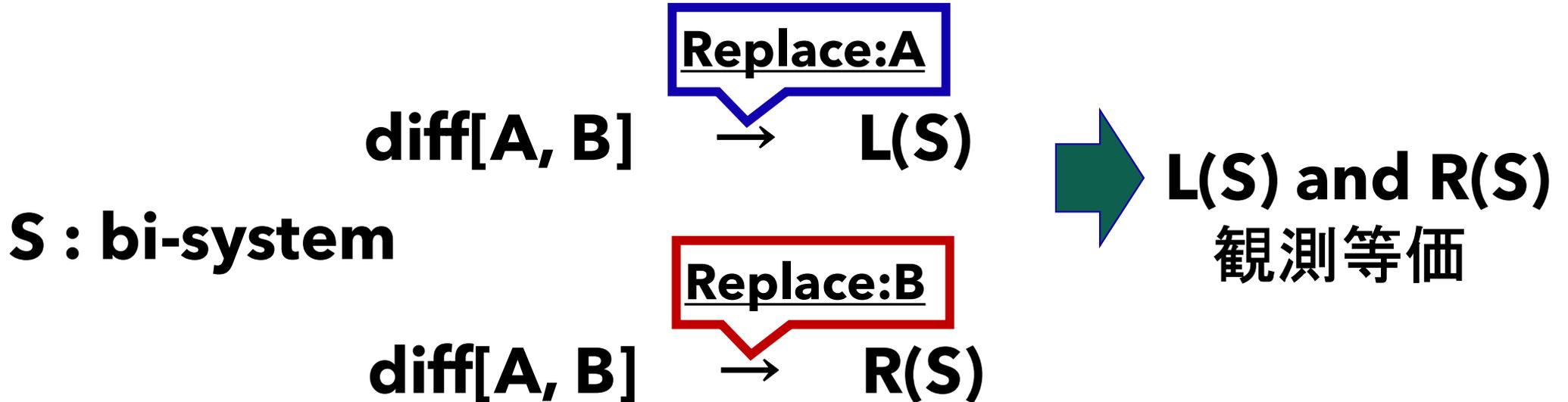
[参考文献.B]

# 観測等価性の証明を自動化 : *Bi-system*

## Bi processes

**diff[-, -] : 多項式書き換えシステム**

2つの実行を「同時に実行する」プロセスを入力し、攻撃者に観測させる  
Ex.  $P(A,B)$ と $P(B,A) \rightarrow P(\text{diff}[A, B], \text{diff}[B, A])$



[参考文献.B]

Blanchet, B., Abadi, M., Fournet, C.: Automated verification of selected equivalences for security protocols. *Journal of Logic and Algebraic Programming* 75(1), 3-51 (2008) 9/35

## 5. 検証概要

1. ProVerif
2. Tamarin-prover



# 項のみが異なるプロセス間の観測等価性: 検証する operator

## #1. ProVerif

**choice[<term>, <term>] ※1**

## #2. Tamarin-prover

**diff (x, y) ※2**

### [補足事項]

※1 : プロセスに choice を含む場合, 「query」, 「noninterf」, 「weaksecret」は使用できない

※2 : ユーザが指定できる「lemma」が無い

# DDH-Code

## ProVerif

```
(*code: decision_diffie_hellman ProVerif*)
type G.
type exponent.
const g: G [data].
free d: channel.

fun exp(G, exponent): G.

equation forall x: exponent, y: exponent;
exp(exp(g,x),y) = exp(exp(g,y),x).

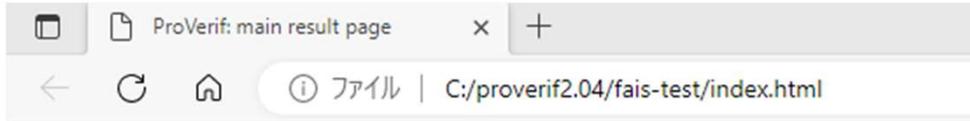
process
new a: exponent; new b: exponent; new c: exponent;
out(d, (exp(g,a), exp(g,b), choice[exp(exp(g,a),b), exp(g,c)])))
```

## Tamarin-prover

```
(*code: decision_diffie_hellman Tamarin-prover*)
theory decision_diffie_hellman
begin
builtins: diffie-hellman
functions: g/0

rule DDH:
[ Fr(~a), Fr(~b), Fr(~c) ] --[ ]->
[ Out(<g^(~a), g^(~b), diff(g^(~a)^(~b),g^(~c))>) ]
end
```

# ProVerif : html出力コマンドを使用し、実施している内容を可視化



## ProVerif results

### [Equations & Destructors](#)

[Biprocess 0](#) (that is, the initial process)

- Observational equivalence in [biprocess 0](#).  
[Clauses](#)  
[Completed clauses](#)  
RESULT Observational equivalence is true.

### Verification summary:

- Observational equivalence is true.

## CUIからのLog出力以外にも実施

### Equations

#### Linear part

Initial equations:

- $\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

Completed equations:

- $\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

#### Convergent part

No equation.

### Completed destructors

```
(fail-any || @mayfail_v) => fail-any
(v || @mayfail_v) => @mayfail_v if v ≠ true
(true || @mayfail_v) => true
```

```
not(fail-any) => fail-any
not(v) => true if v ≠ true
not(true) => false
```

```
is_nat(fail-any) => fail-any
is_nat(v) => false if is_not_nat(v)
is_nat(v) => true if is_nat(v)
```

```
(v ≥ fail-any) => fail-any
(fail-any ≥ @mayfail_v) => fail-any
(v ≥ v_1) => fail-any if is_not_nat(v_1)
(v ≥ v_1) => fail-any if is_not_nat(v)
(v ≥ v_1) => false if v_1 ≥ v + 1
(v ≥ v_1) => true if v ≥ v_1
```

```
(v > fail-any) => fail-any
(fail-any > @mayfail v) => fail-any
```

# Tamarin-prover : interactive mode を使用

## Proof scripts

```
theory decision_diffie_hellman begin
  Diff Rules
  LHS: Message theory
  RHS: Message theory
  LHS: Message theory [Diff]
  RHS: Message theory [Diff]
  LHS: Multiset rewriting rules (3)
  RHS: Multiset rewriting rules (3)
  LHS: Multiset rewriting rules [Diff] (3)
  RHS: Multiset rewriting rules [Diff] (3)
  LHS: Raw sources (4 cases, deconstructions complete)
  RHS: Raw sources (4 cases, deconstructions complete)
  LHS: Raw sources [Diff] (4 cases, deconstructions complete)
  RHS: Raw sources [Diff] (4 cases, deconstructions complete)
  LHS: Refined sources (4 cases, deconstructions complete)
  RHS: Refined sources (4 cases, deconstructions complete)
  LHS: Refined sources [Diff] (4 cases, deconstructions complete)
  RHS: Refined sources [Diff] (4 cases, deconstructions complete)
  LHS: Lemmas
  RHS: Lemmas
  Diff-Lemmas
  lemma Observational_equivalence:
  by sorry
end
```

## Visualization display

Applicable Proof Methods: Goals sorted according to the 'smart' heuristic (for diff proofs)

1. rule-equivalence // Prove equivalence using rule equivalence

- a. autoprove (A. for all solutions)
- b. autoprove (B. for all solutions) with proof-depth bound 5

Constraint system

proof type: none

current rule: none

system: none

mirror system: none

protocol rules:

construction rules:

destruction rules:

0 sub-case(s)

観測等価性の証明をする場合には、“sorry”を押下する

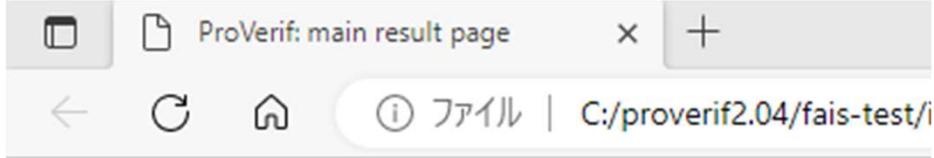
CUIからのLog出力以外にも実施

## 6. 検証の詳細結果

1. ProVerif
2. Tamarin-prover



# ProVerif - verification details



## ProVerif results

### [Equations & Destructors](#)

### [Biprocess 0](#) (that is, the initial process)

- Observational equivalence in [biprocess 0](#).  
[Clauses](#)  
[Completed clauses](#)  
RESULT Observational equivalence is true.

## Verification summary:

- Observational equivalence is true.

(\*log: decision\_diffie\_hellman ProVerif\*)

Linear part:

$\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

Completing equations...

Completed equations:

$\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

Convergent part: No equation.

Biprocess 0 (that is, the initial process):

{1}new a: exponent;

{2}new b: exponent;

{3}new c: exponent;

{4}out(d, (exp(g,a),exp(g,b),choice[exp(exp(g,a),b),exp(g,c)]))

— Observational equivalence in biprocess 0.

Translating the process into Horn clauses...

Termination warning:  $v \neq v_1 \ \&\& \ \text{attacker2}(v_2,v) \ \&\& \ \text{attacker2}(v_2,v_1) \rightarrow \text{bad}$

Selecting 0

Termination warning:  $v \neq v_1 \ \&\& \ \text{attacker2}(v,v_2) \ \&\& \ \text{attacker2}(v_1,v_2) \rightarrow \text{bad}$

Selecting 0

Completing...

Termination warning:  $v \neq v_1 \ \&\& \ \text{attacker2}(v_2,v) \ \&\& \ \text{attacker2}(v_2,v_1) \rightarrow \text{bad}$

Selecting 0

Termination warning:  $v \neq v_1 \ \&\& \ \text{attacker2}(v,v_2) \ \&\& \ \text{attacker2}(v_1,v_2) \rightarrow \text{bad}$

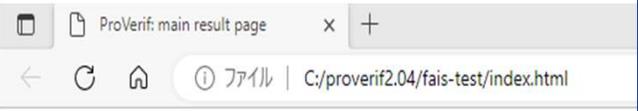
Selecting 0

RESULT Observational equivalence is true.

Verification summary:

Observational equivalence is true.

# ProVerif - verification details



## ProVerif results

[Equations & Destructors](#)

[Biprocess 0](#) (that is, the initial process)

- Observational equivalence in [biprocess 0](#).
  - [Clauses](#)
  - [Completed clauses](#)
- RESULT Observational equivalence is true.

## Verification summary:

- Observational equivalence is true.

## Equations

### Linear part

Initial equations:

- $\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

Completed equations:

- $\text{exp}(\text{exp}(g,x),y) = \text{exp}(\text{exp}(g,y),x)$

### Convergent part

No equation.

## Completed destructors

```
(fail-any || @mayfail_v) => fail-any
(v || @mayfail_v) => @mayfail_v if v ≠ true
(true || @mayfail_v) => true
```

```
not(fail-any) => fail-any
not(v) => true if v ≠ true
not(true) => false
```

```
is_nat(fail-any) => fail-any
is_nat(v) => false if is_not_nat(v)
is_nat(v) => true if is_nat(v)
```

```
(v ≥ fail-any) => fail-any
(fail-any ≥ @mayfail_v) => fail-any
(v ≥ v_1) => fail-any if is_not_nat(v_1)
(v ≥ v_1) => fail-any if is_not_nat(v)
(v ≥ v_1) => false if v_1 ≥ v + 1
(v ≥ v_1) => true if v ≥ v_1
```

## Biprocess 0 (the initial process)

```
{1} new a: exponent;
{2} new b: exponent;
{3} new c: exponent;
{4} out(d, (exp(g,a),exp(g,b),choice[exp(exp(g,a),b),exp(g,c)]))
```

# ProVerif - verification details



## ProVerif results

[Equations & Destructors](#)

[Biprocess 0](#) (that is, the initial process)

- Observational equivalence in [biprocess 0](#).

[Clauses](#)

[Completed clauses](#)

RESULT Observational equivalence is true.

## Verification summary:

- Observational equivalence is true.

例: **clause12**  
攻撃者が関数expを適用

## Completed clauses

- `attacker2(x,x_1) && attacker2(y,y_1) -> attacker2(exp(exp(g,y),x),exp(exp(g,y_1),x_1))`

## Initial clauses

- Clause 0: `v ≠ true && attacker2(v,true) && attacker2(@mayfail_v,@mayfail_v_1) -> attacker2(@mayfail_v,true)`  
(The attacker applies function ||.)
- Clause 1: `v ≠ true && attacker2(true,v) && attacker2(@mayfail_v,@mayfail_v_1) -> attacker2(true,@mayfail_v_1)`  
(The attacker applies function ||.)
- Clause 2: `attacker2(true,true)`  
(The attacker applies function true.)
- Clause 3: `v ≠ true && attacker2(v,true) -> attacker2(true,false)`  
(The attacker applies function not.)
- Clause 4: `v ≠ true && attacker2(true,v) -> attacker2(false,true)`  
(The attacker applies function not.)
- Clause 5: `is_nat(v) && is_not_nat(v_1) && attacker2(v_1,v) -> attacker2(false,true)`  
(The attacker applies function is\_nat.)
- Clause 6: `is_nat(v) && is_not_nat(v_1) && attacker2(v,v_1) -> attacker2(true,false)`  
(The attacker applies function is\_nat.)
- Clause 7: `attacker2(g,g)`  
(The attacker applies function g.)
- Clause 8: `attacker2(false,false)`  
(The attacker applies function false.)
- Clause 9: `attacker2(v,v_1) && attacker2(v_2,v_3) -> attacker2(exp(v,v_2),exp(v_1,v_3))`  
(The attacker applies function exp.)
- Clause 10: `attacker2(v,exp(g,x)) && attacker2(v_1,y) -> attacker2(exp(v,v_1),exp(exp(g,y),x))`  
(The attacker applies function exp.)
- Clause 11: `attacker2(exp(g,x),v) && attacker2(y,v_1) -> attacker2(exp(exp(g,y),x),exp(v,v_1))`  
(The attacker applies function exp.)
- Clause 12: `attacker2(exp(g,x),exp(g,x_1)) && attacker2(y,y_1) -> attacker2(exp(exp(g,y),x),exp(exp(g,y_1),x_1))`  
(The attacker applies function exp.)
- Clause 13: `is_not_nat(v) && v_1 ≥ v_2 + 1 && attacker2(v_3,v_2) && attacker2(v,v_1) -> attacker2(fail-any,false)`  
(The attacker applies function >=.)

# ProVerif - verification details



## ProVerif results

[Equations & Destructors](#)

[Biprocess 0](#) (that is, the initial process)

- Observational equivalence in [biprocess 0](#).

[Clauses](#)

[Completed clauses](#)

RESULT Observational equivalence is true.

## Verification summary:

- Observational equivalence is true.

## Completed clauses

- `attacker2(v,y) -> attacker2(exp(exp(exp(g,a[]),b[]),v),exp(exp(g,y),c[]))`
- `attacker2(v,y) -> attacker2(exp(exp(exp(g,b[]),a[]),v),exp(exp(g,y),c[]))`
- `attacker2(v,y) -> attacker2(exp(exp(g,b[]),v),exp(exp(g,y),b[]))`
- `attacker2(y,v) -> attacker2(exp(exp(g,y),b[]),exp(exp(g,b[]),v))`
- `attacker2(y,y_1) -> attacker2(exp(exp(g,y),b[]),exp(exp(g,y_1),b[]))`
- `attacker2(v,y) -> attacker2(exp(exp(g,a[]),v),exp(exp(g,y),a[]))`
- `attacker2(y,v) -> attacker2(exp(exp(g,y),a[]),exp(exp(g,a[]),v))`
- `attacker2(y,y_1) -> attacker2(exp(exp(g,y),a[]),exp(exp(g,y_1),a[]))`
- `attacker2(x,x_1) && attacker2(y,y_1) -> attacker2(exp(exp(g,y),x),exp(exp(g,y_1),x_1))`
- `attacker2(v,x) && attacker2(v_1,y) -> attacker2(exp(exp(g,v),v_1),exp(exp(g,y),x))`
- `attacker2(exp(exp(g,a[]),b[]),exp(g,c[]))`
- `attacker2(exp(exp(g,b[]),a[]),exp(g,c[]))`
- `attacker2(exp(g,b[]),exp(g,b[]))`
- `attacker2(exp(g,a[]),exp(g,a[]))`
- `attacker2(new-name[!att = v],new-name[!att = v])`
- `attacker2(d[],d[])`
- `attacker2(fail-any,fail-any)`
- `attacker2(v,v_1) -> output2(v,v_1)`
- `attacker2(v,v_1) -> input2(v,v_1)`
- `equal(v,v)`
- `attacker2(v,v_1) && attacker2(v_2,v_3) -> mess2(v,v_2,v_1,v_3)`
- `attacker2(v,v_1) && attacker2(v_2,v_3) && attacker2(v_4,v_5) -> attacker2((v,v_2,v_4),(v_1,v_3,v_5))`
- `attacker2(v,v_1) -> attacker2(v + 1,v_1 + 1)`
- `attacker2(0,0)`
- `attacker2(v,v_1) && attacker2(v_2,v_3) -> attacker2(exp(v,v_2),exp(v_1,v_3))`
- `attacker2(false,false)`
- `attacker2(g,g)`
- `attacker2(true,true)`

```
(*log: decision_diffie_hellman ProVerif*)
Linear part:
exp(exp(g,x),y) = exp(exp(g,y),x)
Completing equations...
Completed equations:
exp(exp(g,x),y) = exp(exp(g,y),x)
Convergent part: No equation.
Biprocess 0 (that is, the initial process):
{1}new a: exponent;
{2}new b: exponent;
{3}new c: exponent;
{4}out(d, (exp(g,a),exp(g,b),choice[exp(exp(g,a),b),exp(g,c)]))
— Observational equivalence in biprocess 0.
Translating the process into Horn clauses...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) → bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) → bad
Selecting 0
Completing...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) → bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) → bad
Selecting 0
RESULT Observational equivalence is true.
-----
Verification summary:
Observational equivalence is true.
-----
```

## 別の検証方法

### ProVerif : Choice を使用しない - equivalence -

#### ProVerif equivalence

```
(*code: decision_diffie_hellman ProVerif for equivalence*)
```

```
type G.
```

```
type exponent.
```

```
const g: G [data].
```

```
free d: channel.
```

```
fun exp(G, exponent): G.
```

```
equation forall x: exponent, y: exponent;
```

```
  exp(exp(g,x),y) = exp(exp(g,y),x).
```

```
equivalence
```

```
  new a: exponent; new b: exponent; new c: exponent; out(d, (exp(g,a), exp(g,b), exp(exp(g,a),b)))
```

```
  new a: exponent; new b: exponent; new c: exponent; out(d, (exp(g,a), exp(g,b), exp(g,c)))
```

```

Linear part:
exp(exp(g,x),y) = exp(exp(g,y),x)
Completing equations...
Completed equations:
exp(exp(g,x),y) = exp(exp(g,y),x)
Convergent part: No equation.
Process 1 (that is, the first initial process):
{1}new a: exponent;
{2}new b: exponent;
{3}new c: exponent;
{4}out(d, (exp(g,a),exp(g,b),exp(exp(g,a),b)))

Process 2 (that is, the second initial process):
{5}new a_1: exponent;
{6}new b_1: exponent;
{7}new c_1: exponent;
{8}out(d, (exp(g,a_1),exp(g,b_1),exp(g,c_1)))

```

```

— Observational equivalence between two processes
— Observational equivalence in biprocess 3 (that is, the merge of process 1 and process 2, simplified)
{1}new a_2: exponent;
{2}new b_2: exponent;
{3}new a_3: exponent;
{4}new b_3: exponent;
{5}new c_2: exponent;
{6}out(d, choice [(exp(g,a_2),exp(g,b_2),exp(exp(g,a_2),b_2)) . (exp(g,a_3),exp(g,b_3),exp(g,c_2))])

```

```

Translating the process into Horn clauses...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) -> bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) -> bad
Selecting 0
Completing...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) -> bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) -> bad
Selecting 0
RESULT Observational equivalence is true.

```

Verification summary:

Equivalence between process 1 and process 2 is true.

# Result

## Choiceを使用

```
(*log: decision_diffie_hellman ProVerif*)
Linear part:
exp(exp(g,x),y) = exp(exp(g,y),x)
Completing equations...
Completed equations:
exp(exp(g,x),y) = exp(exp(g,y),x)
Convergent part: No equation.
Biprocess 0 (that is, the initial process):
{1}new a: exponent;
{2}new b: exponent;
{3}new c: exponent;
{4}out(d, (exp(g,a),exp(g,b),choice[exp(exp(g,a),b),exp(g,c)]))
— Observational equivalence in biprocess 0.
Translating the process into Horn clauses...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1)
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2)
Selecting 0
Completing...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1)
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2)
Selecting 0
RESULT Observational equivalence is true.
```

---

```
Verification summary:
Observational equivalence is true.
```

---

# ProVerif - verification details

## equivalenceを使用

### equivalence Log ProVerif

```
Linear part:
exp(exp(g,x),y) = exp(exp(g,y),x)
Completing equations...
Completed equations:
exp(exp(g,x),y) = exp(exp(g,y),x)
Convergent part: No equation.
Process 1 (that is, the first initial process):
{1}new a: exponent;
{2}new b: exponent;
{3}new c: exponent;
{4}out(d, (exp(g,a),exp(g,b),exp(exp(g,a),b)))

Process 2 (that is, the second initial process):
{5}new a_1: exponent;
{6}new b_1: exponent;
{7}new c_1: exponent;
{8}out(d, (exp(g,a_1),exp(g,b_1),exp(g,c_1)))

— Observational equivalence between two processes
— Observational equivalence in biprocess 3 (that is, the merge of process 1 and process 2, simplified)
{1}new a_2: exponent;
{2}new b_2: exponent;
{3}new a_3: exponent;
{4}new b_3: exponent;
{5}new c_2: exponent;
{6}out(d, choice[(exp(g,a_2),exp(g,b_2),exp(exp(g,a_2),b_2)), (exp(g,a_3),exp(g,b_3),exp(g,c_2))])

Translating the process into Horn clauses...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) → bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) → bad
Selecting 0
Completing...
Termination warning: v ≠ v_1 && attacker2(v_2,v) && attacker2(v_2,v_1) → bad
Selecting 0
Termination warning: v ≠ v_1 && attacker2(v,v_2) && attacker2(v_1,v_2) → bad
Selecting 0
RESULT Observational equivalence is true.
```

---

```
Verification summary:
```

```
Equivalence between process 1 and process 2 is true.
```

---

# Tamarin-prover - verification details

# CUI実行Log

```
(*log: decision_diffie_hellman Tamarin-prover *)
```

```
:  
(omitted)
```

```
rule (modulo E) DDH:
```

```
[ Fr( ~a ), Fr( ~b ), Fr( ~c ) ]  $\longrightarrow$   
[ Out( <g^~a, g^~b, diff(g^~a^~b, g^~c)> ) ]
```

```
/* All well-formedness checks were successful. */
```

```
diffLemma Observational_equivalence:
```

```
rule-equivalence
```

```
case Rule_DDH
```

```
backward-search
```

```
case LHS
```

```
step( simplify )
```

```
MIRRORED
```

```
next
```

```
case RHS
```

```
step( simplify )
```

```
MIRRORED
```

```
qed
```

```
next
```

```
case Rule_Destr_d_0fst
```

```
backward-search
```

```
case LHS
```

```
step( simplify )
```

```
next  
case pair  
MIRRORED  
qed  
qed  
qed  
next  
:  
(omitted)  
:  
next  
case Rule_Send  
backward-search  
case LHS  
step( simplify )  
MIRRORED  
next  
case RHS  
step( simplify )  
MIRRORED  
qed  
qed  
end
```

---

```
summary of summaries:
```

```
analyzed: fais-tamarin-ddh-20230113.spthy
```

```
DiffLemma: Observational_equivalence : verified (2522 steps)
```

---

# Tamarin-prover - verification details

## Proof scripts

```
theory decision_diff_theorem begin
```

```
Diff Rules
```

```
LHS: Message theory
```

```
RHS: Message theory
```

```
LHS: Message theory [Diff]
```

```
RHS: Message theory [Diff]
```

```
LHS: Multiset rewriting rules (3)
```

```
RHS: Multiset rewriting rules (3)
```

```
LHS: Multiset rewriting rules [Diff] (3)
```

```
RHS: Multiset rewriting rules [Diff] (3)
```

```
LHS: Raw sources (4 cases, deconstructions complete)
```

```
RHS: Raw sources (4 cases, deconstructions complete)
```

```
LHS: Raw sources [Diff] (4 cases, deconstructions complete)
```

```
RHS: Raw sources [Diff] (4 cases, deconstructions complete)
```

```
LHS: Refined sources (4 cases, deconstructions complete)
```

```
RHS: Refined sources (4 cases, deconstructions complete)
```

```
LHS: Refined sources [Diff] (4 cases, deconstructions complete)
```

```
RHS: Refined sources [Diff] (4 cases, deconstructions complete)
```

```
LHS: Lemmas
```

```
RHS: Lemmas
```

```
Diff-Lemmas
```

```
lemma Observational_equivalence:  
by sorry
```

```
end
```

## Visualization display

Applicable Proof Methods: Goals sorted according to the 'smart' heuristic (for diff proofs)

1. **rule-equivalence** // Prove equivalence using rule equivalence

a. **autoprove** (A. for all solutions)

b. **autoprove** (B. for all solutions) with proof-depth bound 5

Constraint system

proof type: none

current rule: none

system: none

mirror system: none

protocol rules:

construction rules:

destruction rules:

0 sub-case(s)

証明を続ける為に、  
"rule-equivalence"  
押下する

2つのサブルーチンを生成する  
それぞれの側で、全ての可能な  
実行を構築する

```
Diff-Lemmas
```

```
lemma Observational_equivalence:  
rule-equivalence  
  case Rule_DDH  
  backward-search  
  case LHS  
  step(simplify)  
  MIRRORED  
  next  
  case RHS  
  step(simplify)  
  MIRRORED  
  qed  
next
```

# Tamarin-prover - verification details

| Proof scripts   | Visualization display  |
|---|--|
| <pre> RHS: Multiset rewriting rules [Diff] (3) LHS: Raw sources (4 cases, deconstructions complete) RHS: Raw sources (4 cases, deconstructions complete) LHS: Raw sources [Diff] (4 cases, deconstructions complete) RHS: Raw sources [Diff] (4 cases, deconstructions complete) LHS: Refined sources (4 cases, deconstructions complete) RHS: Refined sources (4 cases, deconstructions complete) LHS: Refined sources [Diff] (4 cases, deconstructions complete) RHS: Refined sources [Diff] (4 cases, deconstructions complete) LHS: Lemmas RHS: Lemmas  Diff-Lemmas lemma Observational_equivalence: rule-equivalence case Rule_DDH backward-search case LHS step(simplify) MIRRORED next case RHS step(simplify) MIRRORED qed next case Rule_Destr_d_fst by sorry next case Rule_Destr_d_snd by sorry next case Rule_Destr_exp by sorry next case Rule_Destr_inv by sorry next case Rule_Equality by sorry next case Rule_Send by sorry qed end </pre> | <pre> Applicable Proof Methods: Goals sorted according to the 'smart' heuristic 1. backward-search // Do backward search from rule a. autoprove (A. for all solutions) b. autoprove (B. for all solutions) with proof-depth bound 5  Constraint system proof type: RuleEquivalence current rule: IntrDestr_0_fst system: none mirror system: none  protocol rules: rule (modulo E) DDH: [ Fr( ~a ), Fr( ~b ), Fr( ~c ) ] -&gt; [ Out( &lt;g^~a, g^~b, diff(g^~a~b, g^~c)&gt; ) ]  construction rules: rule (modulo AC) c_exp: [ !KU( x ), !KU( x.1 ) ]--[ !KU( x^x.1 ) ]-&gt; [ !KU( x^x.1 ) ]  rule (modulo AC) c_fst: [ !KU( x ) ]--[ !KU( fst(x) ) ]-&gt; [ !KU( fst(x) ) ]  rule (modulo AC) c_g: [ ]--[ !KU( g ) ]-&gt; [ !KU( g ) ]  rule (modulo AC) c_inv: [ !KU( x ) ]--[ !KU( inv(x) ) ]-&gt; [ !KU( inv(x) ) ]  rule (modulo AC) c_mult: [ !KU( x ), !KU( x.1 ) ] --[ !KU( &lt;x, x.1&gt; ) ]-&gt; [ !KU( x*x.1 ) ]  rule (modulo AC) c_one: [ ]--[ !KU( one ) ]-&gt; [ !KU( one ) ]  rule (modulo AC) c_pair: [ !KU( x ), !KU( x.1 ) ] --[ !KU( &lt;x, x.1&gt; ) ]-&gt; [ !KU( &lt;x, x.1&gt; ) ]  rule (modulo AC) c_snd: [ !KU( x ) ]--[ !KU( snd(x) ) ]-&gt; [ !KU( snd(x) ) ] </pre> |

construction rules:

rule (modulo AC) c\_exp:  
 $[ !KU( x ), !KU( x.1 ) ] \text{--}[ !KU( x^x.1 ) ] \text{--}> [ !KU( x^x.1 ) ]$

rule (modulo AC) c\_fst:  
 $[ !KU( x ) ] \text{--}[ !KU( fst(x) ) ] \text{--}> [ !KU( fst(x) ) ]$

rule (modulo AC) c\_g:  
 $[ ] \text{--}[ !KU( g ) ] \text{--}> [ !KU( g ) ]$

rule (modulo AC) c\_inv:  
 $[ !KU( x ) ] \text{--}[ !KU( inv(x) ) ] \text{--}> [ !KU( inv(x) ) ]$

rule (modulo AC) c\_mult:  
 $[ !KU( x ), !KU( x.1 ) ] \text{--}[ !KU( <x, x.1> ) ] \text{--}> [ !KU( x*x.1 ) ]$

rule (modulo AC) c\_one:  
 $[ ] \text{--}[ !KU( one ) ] \text{--}> [ !KU( one ) ]$

rule (modulo AC) c\_pair:  
 $[ !KU( x ), !KU( x.1 ) ] \text{--}[ !KU( <x, x.1> ) ] \text{--}> [ !KU( <x, x.1> ) ]$

rule (modulo AC) c\_snd:  
 $[ !KU( x ) ] \text{--}[ !KU( snd(x) ) ] \text{--}> [ !KU( snd(x) ) ]$

# Tamarin-prover - verification details

## Isend

攻撃者が知っている値 $x$ を取り、それをプロトコルの入力 $In(x)$ に渡している

### Proof scripts

```
theory decision_diffie_hellman begin
```

```
Diff Rules
```

```
LHS: Message theory
```

```
RHS: Message theory
```

```
LHS: Message theory [Diff]
```

```
RHS: Message theory [Diff]
```

```
LHS: Multiset rewriting rules (3)
```

## Irecv

プロトコルの出力 $Out(x)$ を受け取り、  
攻撃者! $KD(x)$ に渡している。

### Multiset rewriting rules and restrictions [LHS]

#### Multiset Rewriting Rules

```
rule (modulo AC) isend:  
  [ !KU( x ) ] --[ K( x ) ]-> [ In( x ) ]
```

```
rule (modulo AC) irecv:  
  [ Out( x ) ] --> [ !KD( x ) ]
```

```
rule (modulo AC) DDH:  
  [ Fr( ~a ), Fr( ~b ), Fr( ~c ) ]  
  -->  
  [ Out( <g^~a, g^~b, g^(~a*~b)> ) ]
```

### Multiset rewriting rules and restrictions [RHS]

#### Multiset Rewriting Rules

```
rule (modulo AC) isend:  
  [ !KU( x ) ] --[ K( x ) ]-> [ In( x ) ]
```

```
rule (modulo AC) irecv:  
  [ Out( x ) ] --> [ !KD( x ) ]
```

```
rule (modulo AC) DDH:  
  [ Fr( ~a ), Fr( ~b ), Fr( ~c ) ]  
  -->  
  [ Out( <g^~a, g^~b, g^~c> ) ]
```

攻撃者が $(g^a, g^b, g^{ab})$ は  
 $(g^a, g^b, g^c)$ と、計算上区別がつかない

# Tamarin-prover - verification details

## Proof scripts

```

RHS: Multiset rewriting rules [Diff] (3)
LHS: Raw sources (4 cases, deconstructions complete)
RHS: Raw sources (4 cases, deconstructions complete)
LHS: Raw sources [Diff] (4 cases, deconstructions complete)
RHS: Raw sources [Diff] (4 cases, deconstructions complete)
LHS: Refined sources (4 cases, deconstructions complete)
RHS: Refined sources (4 cases, deconstructions complete)
LHS: Refined sources [Diff] (4 cases, deconstructions complete)
RHS: Refined sources [Diff] (4 cases, deconstructions complete)
LHS: Lemmas
RHS: Lemmas

Diff-Lemmas
lemma Observational_equivalence:
rule-equivalence
case Rule_DDH
backward-search
case LHS
step(simplify)
MIRRORED
next
case RHS
step(simplify)
MIRRORED
qed

```

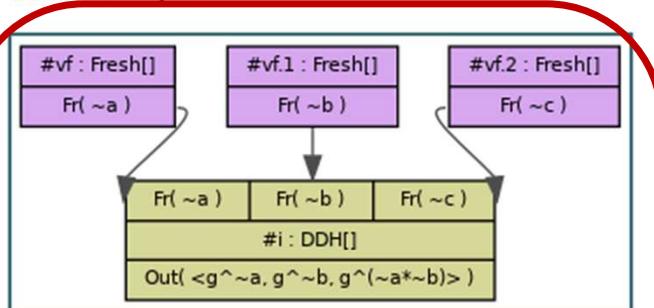
## Method: MIRRORED ...

Applicable Proof Methods: Goals sorted according to the 'smart' heuristic (for diff proofs)

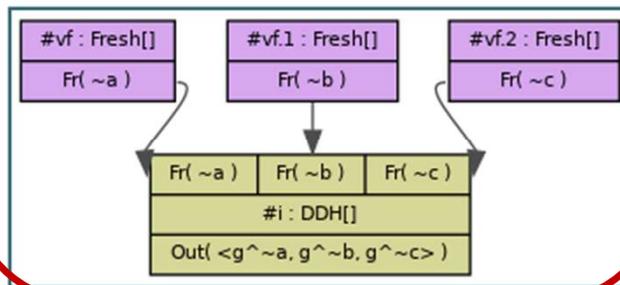
1. **MIRRORED** // Backward search completed

- autoprove (A. for all solutions)
- autoprove (B. for all solutions) with proof-depth bound 5

### Constraint system



### mirror:



proof type: RuleEquivalence

current rule: ProtoDDH

system:  
last: none

formulas:

equations:  
subst:  
conj:

lemmas:

allowed cases: refined

solved formulas:  $\exists \#i. (\text{DiffProtoDDH}() @ \#i)$

unsolved goals:

solved goals:  
DiffProtoDDH() @ #i // nr: 0 (from rule DDH)" (useful2)"

mirror system:

nodes:  
#i: instance (modulo AC) DDH:  
[ Fr(~a), Fr(~b), Fr(~c) ]  
->  
[ Out(<g^~a, g^~b, g^~c> ) ]  
#vf: instance (modulo AC) Fresh:  
[ ] -> [ Fr(~a) ]  
#vf.1: instance (modulo AC) Fresh:  
[ ] -> [ Fr(~b) ]  
#vf.2: instance (modulo AC) Fresh:  
[ ] -> [ Fr(~c) ]

actions:

edges:  
(#vf, 0) >-> (#i, 0), (#vf.1, 0) >-> (#i, 1),  
(#vf.2, 0) >-> (#i, 2)

less:

unsolved goals:  
last: none

formulas:

# Tamarin-prover - verification details

Method: MIRRORED ...

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*x.5) ), IKU( inv(x.2*x.3) ) ]
  ->
  [ IKD( x.4^(x.5*inv(x.2)) ) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*x.5) ), IKU( x.2*inv(x.3) ) ]
  ->
  [ IKD( x.4^(x.2*x.5) ) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*x.5*inv(x.2)) ), IKU( (x.2*inv(x.3)) ) ]
  ->
  [ IKD( x.4^x.5 ) ]
```

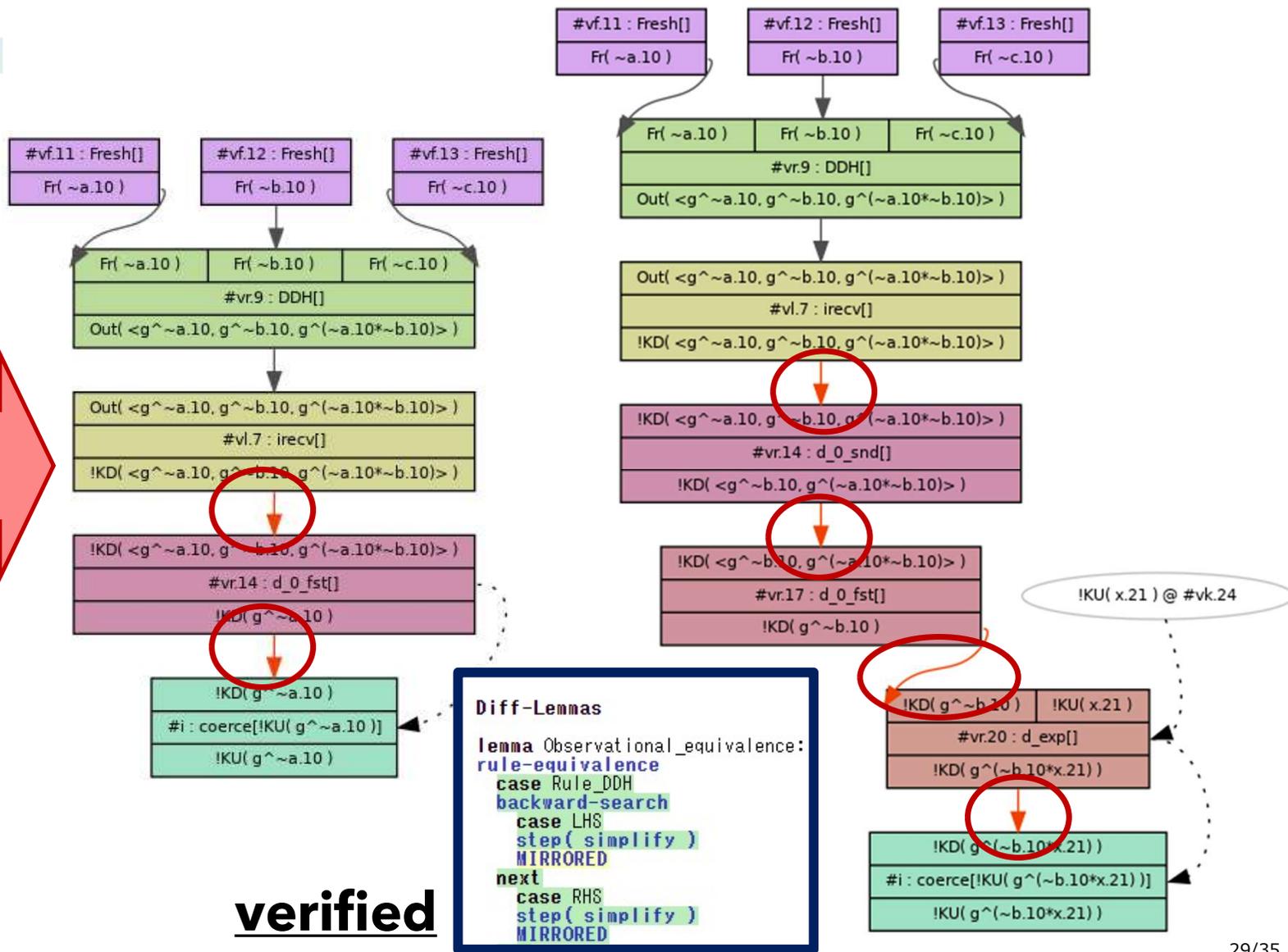
```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*x.5*inv(x.6)) ), IKU( inv(x.2*x.3) ) ]
  ->
  [ IKD( x.4^(x.5*inv(x.2*x.6))) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*x.5*inv(x.2*x.6)) ), IKU( (x.2*inv(x.3)) ) ]
  ->
  [ IKD( x.4^(x.5*inv(x.6)) ) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*inv(x.2)) ), IKU( (x.2*inv(x.3)) ) ]
  ->
  [ IKD( x.4 ) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*inv(x.5)) ), IKU( inv(x.2*x.3) ) ]
  ->
  [ IKD( x.4*inv(x.2*x.5)) ]
```

```
rule (modulo AC) d_exp:
  [ IKD( x.4^(x.3*inv(x.2*x.5)) ), IKU( (x.2*inv(x.3)) ) ]
  ->
  [ IKD( x.4*inv(x.5) ) ]
```



## 別の検証方法

## "restriction" も使用

```
(*code: decision_diffie_hellman Tamarin-prover*)
theory decision_diffie_hellman_restriction
begin
builtins: diffie-hellman
functions: g/0

restriction Equality:
  "All x y #i. Eq(x,y)@#i ==> x=y"

rule DDH_restriction:
  [ Fr(~a), Fr(~b), Fr(~c) ] --[Eq((g^(~a)^(~b)),g^(~c)) ]->
  [ Out(<g^(~a), g^(~b), diff(g^(~a)^(~b),g^(~c))>) ]
end
```

DDHプロトコルの場合は、 $g^{ab}$ と $g^c$ が $=$ になる場合を除く

観測等価性はユーザが指定できるlemmaが無い  
"restriction"を使って状態空間を取り除く

# Result

## Tamarin-prover - verification details

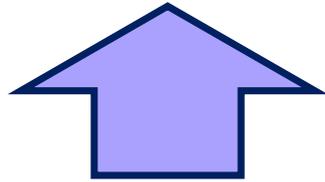
---

---

summary of summaries:  
analyzed: ddh-tamarin-diff\_20230220\_restriction.spthy  
DiffLemma: Observational\_equivalence **verified (124 steps)**

---

---



---

---

summary of summaries:  
analyzed: fais-tamarin-ddh-20230113.spthy  
DiffLemma: Observational\_equivalence : **verified (2522 steps)**

---

---

**DDHプロトコルの場合は、  
diff オペレータをそのまま使用するのではなく、  
restriction を追加で使用した方が良い**

```
(*log: decision_diffie_hellman restriction Tamarin-prover*)
```

```
(complete)
builtins: fst/1, g/0, pair/2, snd/1
equations: fst(<x.1, x.2>) = x.1, snd(<x.1, x.2>) = x.2
```

```
restriction Equality [right]: $\forall$ 
" x y #i. (Eq( x, y ) @ #i)  $\Rightarrow$  (x = y)"
// safety formula
```

```
ty [left]: $\forall$ 
, y ) @ #i)  $\Rightarrow$  (x = y)"
a
```

```
DH_test:
 $\neg$ b ), Fr(  $\neg$ c ) ]
g $\neg$ c ) ]  $\rightarrow$ 
 $\neg$ b, diff(g $\neg$ a $\neg$ b, g $\neg$ c)> ) ]
```

```
iness checks were successful. */
```

```
diffLemma Observational_equivalence:
rule-equivalence
case Rule_DDH_test
backward-search
case LHS
by step( simplify )
next
case RHS
by step( simplify )
qed
```

```
1_0fst
```

```
( !KD( <x, x.1> )  $\blacktriangle$ .0 #i ) )
```

```
( !KD( <x, x.1> )  $\blacktriangle$ .0 #i ) )
```

```
by step( solve( !KD( <x, x.1> )  $\blacktriangle$ .0 #i ) )
qed
next
case Rule_Destr0_snd
backward-search
case LHS
step( simplify )
by step( solve( !KD( <x, x.1> )  $\blacktriangle$ .0 #i ) )
next
case RHS
step( simplify )
by step( solve( !KD( <x, x.1> )  $\blacktriangle$ .0 #i ) )
qed
```

```
(omitted)
:
```

```
next
case Rule_Send
backward-search
case LHS
step( simplify )
MIRRORED
next
case RHS
step( simplify )
MIRRORED
qed
end
```

---

---

summary of summaries:  
analyzed: ddh-tamarin-diff\_20230220\_restriction.spthy  
DiffLemma: Observational\_equivalence : verified (124 steps)

---

---

## 7. 結果の考察

1. ProVerif
2. Tamarin-prover

# 結果の考察

# 検証時間とLogサイズ

| Tool Name      | Operator | Time   | Size of Log |
|----------------|----------|--------|-------------|
| ProVerif       | choice   | 早い     | 少ない         |
| Tamarin-prover | diff     | 時間がかかる | 多い          |



| Tool Name      | Operator    | Time  | Size of Log |
|----------------|-------------|-------|-------------|
| ProVerif       | equivalence | 遅くなる↓ | 増える↑        |
| Tamarin-prover | restriction | 早い↑   | 少ない↑        |

## 8. まとめと今後の課題

- Tamarin-prover : diff は, 厳密な観測等価性を検証するには向いているが, プロトコルによっては restriction で制限をかける方が効率的 (DDH プロトコルの場合)
- ProVerif: 用途に応じて使うクエリの場合分けが必要
  - # モデル検査ツール ProVerif による形式化技法と比較, 記述能力や検証結果の違いを更に明確にする
  - # モジュールへの安全性検証に使用できる, ツールの優位性や使い分けなどを探る

ご清聴ありがとうございました。

