COMPUTATIONAL INDISTINGUISHABILTY AND OBSERVATIONAL EQUIVALENCE

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MOTIVATION

B: I want to sell my product. I am sure that the protocol is secure, however my client requires a proof. What should I do ?

R: Look, here is the definition of universal composability, simply show that your protocol satisfies the property

B: (after weighing up the document): can you do that for me ?

R: no way, it would take me 6 months and I have no time

B: 6 months ? But my product should be on the market next week !

How is it possible to deliver satisfactory proofs (or attacks) within short delays ?

SYMBOLIC VS. COMPUTATIONAL MODELS

	Computational model	Symbolic model
messages	bitstrings	abstract expressions
agents ‖ intruder	Network of Randomized Turing machines	process algebra
Attacker computation power	Any polynomial time computable function	A fixed set of function symbols
Security	Asymptotic, probabilistic: for any attacker, if the keys are large enough, it is very unlikely to break the security	Absolute: no probabilities, unconditional

The Challenge

Symbolic security $+$ Security of primitives $\Rightarrow$ Computational security

Split the problem and

Get the best of the two worlds !

COMPUTATIONAL INTERPRETATION AND PARSING

Comp. interpretation

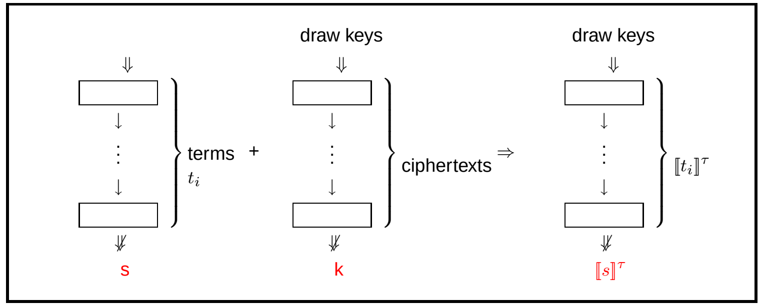
$\longrightarrow$

names $k, k_1, ...$	draw random numbers $\tau(k), \tau(k_1), \ldots$
function symbols: f, $\{\cdot\}^{\cdot}_{\cdot}$, $< \cdot, \cdot >, \ldots$	Actual PTIME computable functions $[\![f]\!], \mathcal{E}, \ldots$
terms $t, f(t_1, \ldots, t_n)$	$[\![f(t_1, \ldots, t_n)]\!]^\tau = [\![f]\!]([\![t_1]\!]^\tau, \ldots, [\![t_n]\!]^\tau)$
predicate symbols: p, typing, =,...	$[\![p]\!]([\![t_1]\!]^\tau, \ldots [\![t_n]\!]^\tau) = p(t_1, ..., t_n)$ with overwhelming probability
(Simple) Processes	Communicating Turing machines

$\longleftarrow$

parsing

WHICH NOTIONS OF SECURITY ?



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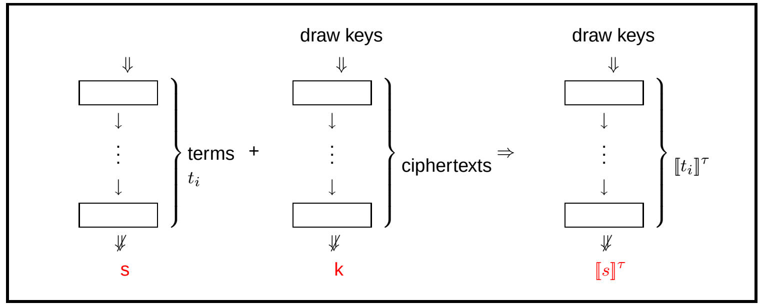

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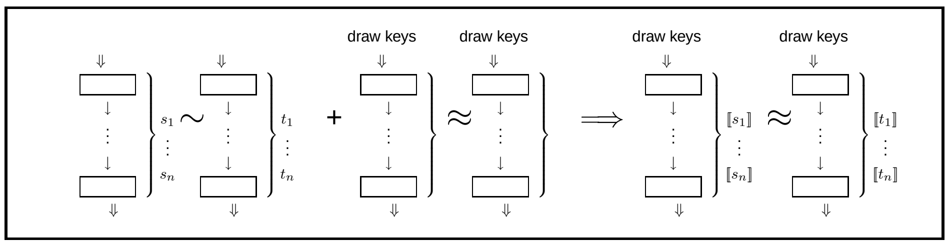

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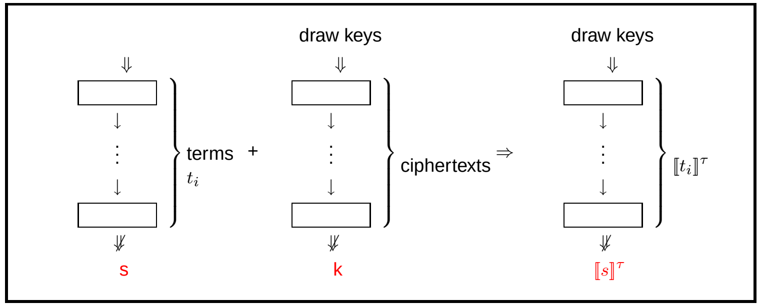

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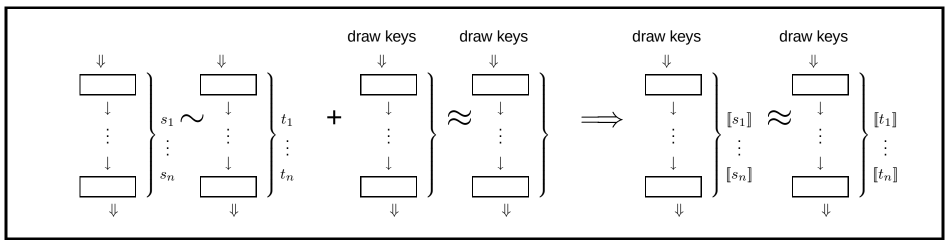

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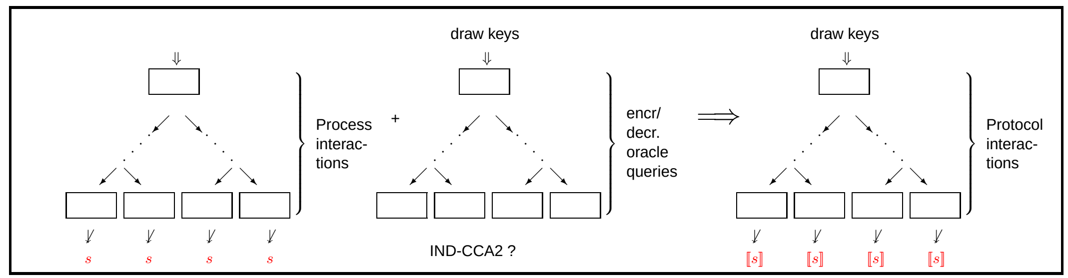

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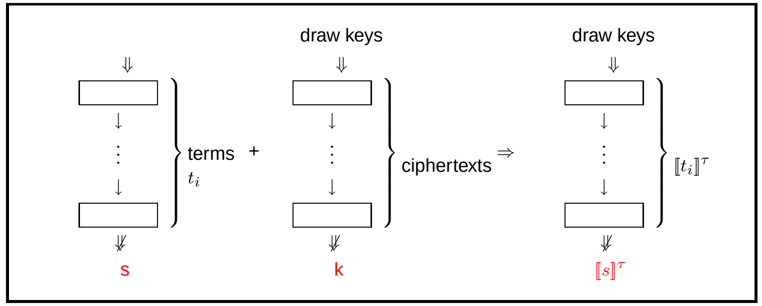

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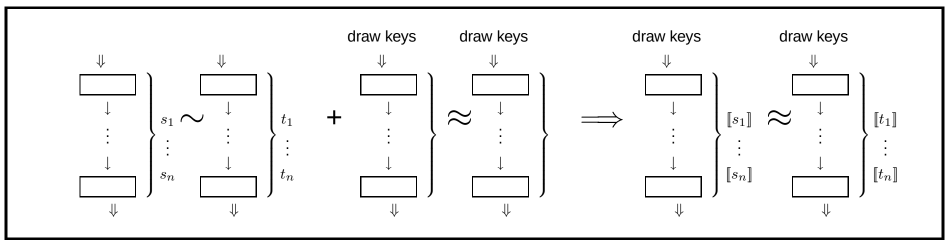

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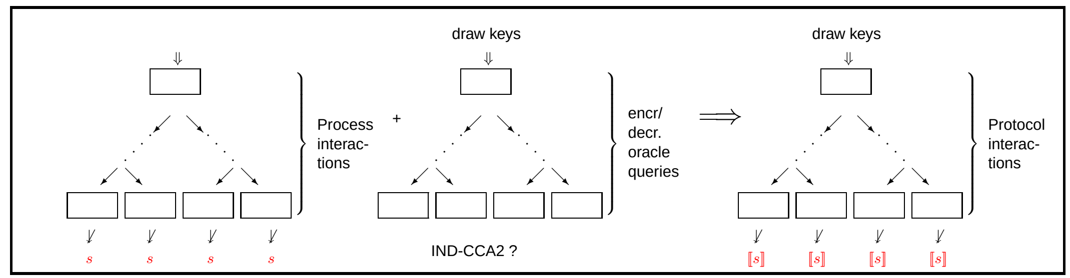

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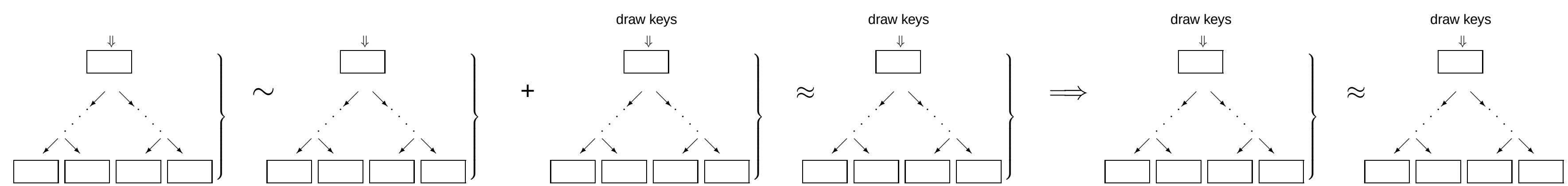

Observational equivalence

Security of primitives: IND-CPA, INT-CTXT...

Active attacks & Indistinguishability; Universal composability

THE CHALLENGE (NEW FORMULATION)

Universal composability vs. Observational equivalence

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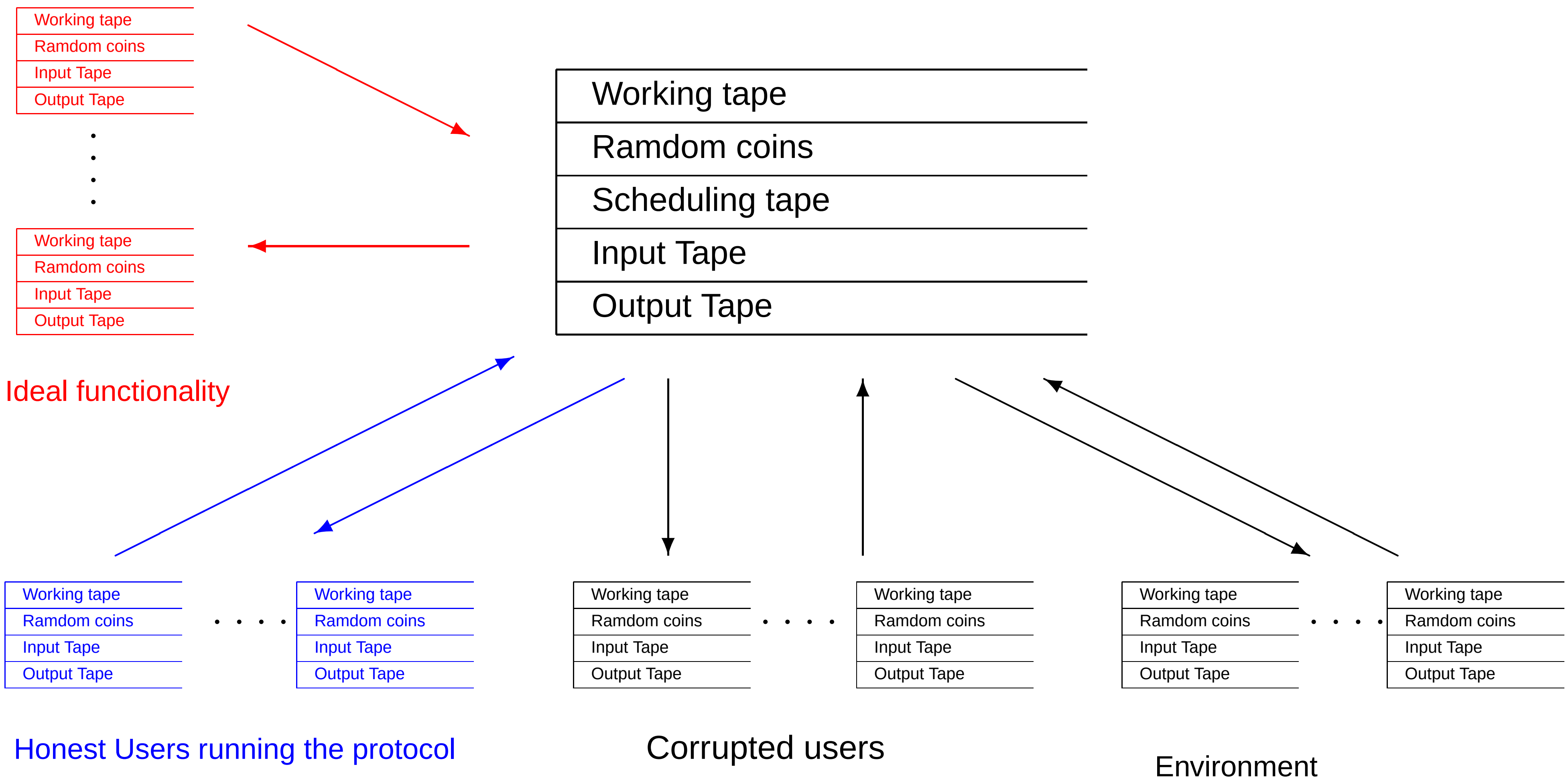
Interactive Turing Machines



All machines run in polynomial time

COMPUTATIONAL INDISTINGUISHABILITY

$\mathcal{F}, \mathcal{F}'$ are two networks of machines (including corrupted ones). $\mathcal{A}$ an interactive Turing machine restricted to work in probabilistic polynomial time.

$$\forall P, \forall \mathcal{A}, \exists N, \forall \eta > N.$$
$$|\mathbf{Pr}\{\overline{r}, r : (\mathcal{A}(r) \| \mathcal{F}(\overline{r}))(0^\eta) = 1\} - \mathbf{Pr}\{\overline{r}, r : (\mathcal{A}(r) \| \mathcal{F}'(\overline{r}))(0^\eta) = 1\}| < \frac{1}{P(\eta)}$$

Typical examples of security properties, which are not trace properties:

- “real or random”: $\mathcal{F}'$ is identical to $\mathcal{F}$ except that confidential value(s) shared by honest agents are replaced by random numbers.
- Anonimity
- guessing attacks
- ...

THE APPLIED π-CALCULUS

$Processes(P, Q)$	$::=$	$P \| Q$	Parallel composition
		$P!$	replication
		$\nu n.P$	n is a new name: scoping
		$c(x).P$	
		$\overline{c}(M) \cdot P$	Terms M build with a fixed set of primitives
		if C then P else Q	

Comm $\quad c(x) \cdot P \| \overline{c}(M) \cdot Q \quad \rightarrow \quad P\{x \mapsto M\} \| Q$

Computation via term rewriting, e.g. dec$($enc$(x, k), k) \rightarrow x$.
Rules for name scoping
Observational equivalence:

$$P \sim_o Q \quad \Longleftrightarrow \quad \forall I.((P \| I) \downarrow_c \Leftrightarrow (Q \| I) \downarrow_c)$$

SIMPLE PROCESSES

$$\nu n_1, \ldots n_k . B_1 \| \cdots \| B_n$$

Each B_i:

$$B ::= \forall \overline{x}.\nu \overline{n}.c(x) \cdot \text{ if } C \text{ then } \overline{c}(M) \cdot B' \text{ else } \mathbf{0}$$

Computational interpretation $[\![B]\!]$:

1. Draw the random values
2. Wait for input: this binds the variable x
3. check the condition (using the computational interpretations and bindings)
4. Send $[\![M]\!]$
5. $\cdots$

A SOUNDNESS RESULT

We assume a *key hierarchy*, a *parsing algorithm*, symmetric keys.

Theorem: If the encryption scheme is joint IND-CPA and INT-CTXT then, for any simple processes P, Q,

$$P \sim_o Q \Rightarrow [\![P]\!] \approx [\![Q]\!]$$

Definition [Bellare & Namprempre, ASIACRYPT 2000] A symmetric encryption scheme is INT-CTXT if, for any non-null polynomial P with positive integer coefficients, for every PPT A with one oracle, there is a N such that, for all $\eta > N$,

$$\mathbf{Pr}\{k \stackrel{R}{\longleftarrow} \mathcal{K}_G(\eta), r, \overline{r} \stackrel{R}{\longleftarrow} U : \exists x, r'. A^{\mathcal{O}_k}(0^\eta \mid r) = \{x\}_k^{r'} \wedge x \notin S\} \leq \frac{1}{P(\eta)}$$

where $S = \{x_1, \ldots, x_n\}$ is the sequence of requests to the oracle, $\mathcal{O}_k(x_i) = \{x_i\}_k^{r_i}$ and $\overline{r}$ is the sequence $(r_1, \ldots, r_n)$.

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$T_P \approx T_Q \Rightarrow [\![P]\!] \approx [\![Q]\!]$ any computational trace is, with an overwhelming probability, an instance of a symbolic trace.

KEY LEMMAS

$$
\begin{array}{rcll}
\Psi_k(n) & = & n & \text{if } n \text{ is a name or a constant} \\
\Psi_k(< t_1, t_2 >) & = & < \Psi_k(t_1), \Psi_k(t_2) > & \\
\Psi_k(\{t\}_k^r) & = & \{0^{l(u)}\}_k^r & \\
\Psi_k(\{t\}_{k'}^r) & = & \{\Psi_k(t)\}_{k'}^r & \text{if } k \neq k'
\end{array}
$$

Ψ_k is extended to computation trees: the adversary is given a view of the execution where any encryption by k has been replaced by encryption of zeros by k.

Lemma: For any computation tree T and for any name k such that k is not deducible from T, $T \sim \Psi_k(T)$.

Lemma: Let P_1 and P_2 be two simple processes such that each P_i admits a key ordering. Let T_{P_i} be the process computation tree associated to P_i. If $T_{P_1} \sim T_{P_2}$ then $T_{P_1} \approx T_{P_2}$ or the encryption scheme is not joint IND-CPA and INT-CTXT.

OPEN QUESTIONS AND FUTURE WORK

Easy:

- Only symmetric encryption –> more primitives
- Replication vs. arbitrary number of processes

Hard:

- Key cycles and key hierarchy
- No dynamic key disclosure (see also [Backes & Pfitzmann 2004])
- Forged keys must be requested to a certification authority (see also [Canetti & Herzog 2006])

Prospective:

- Worst case vs average case attacker
- Getting away from the asymptotic security