Formal Verification of Security Protocols using Automata

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Verification of Security Protocols

Our Question

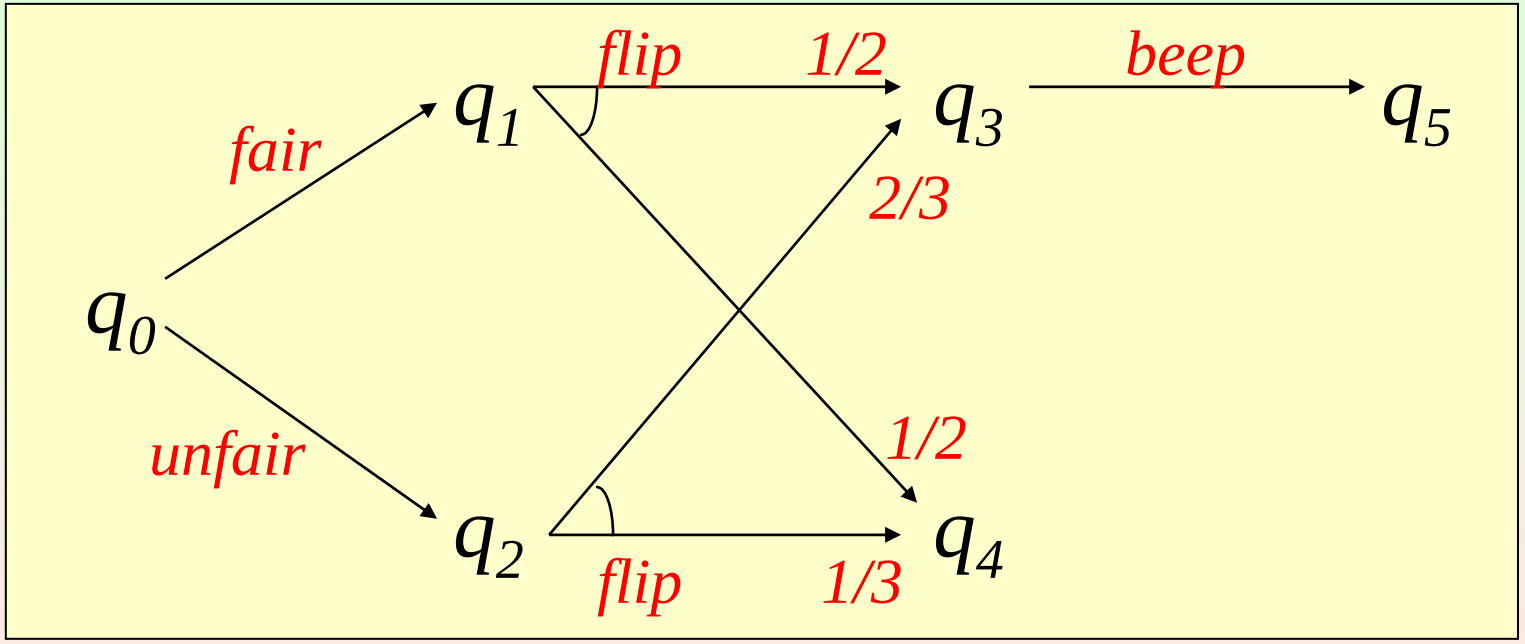
- Can we use Probabilistic Automata?
  - Hierarchical verification
  - Compositional analysis
  - Simulation method
    - Local arguments to derive global properties
    - Rigorous proofs
  - Potentials for automatic verification
  - Potentials to draw connections to other areas

Probabilistic Automata

$$PA = (Q\ ,\ q_0\ ,\ E\ ,\ H\ ,\ D)$$

- Transition relation
  $D \subseteq Q \times (E \cup H) \times \mathrm{Disc}(Q)$
- Internal (hidden) actions
- External actions: $E \cap H = \emptyset$
- Initial state: $q_0 \in Q$
- States

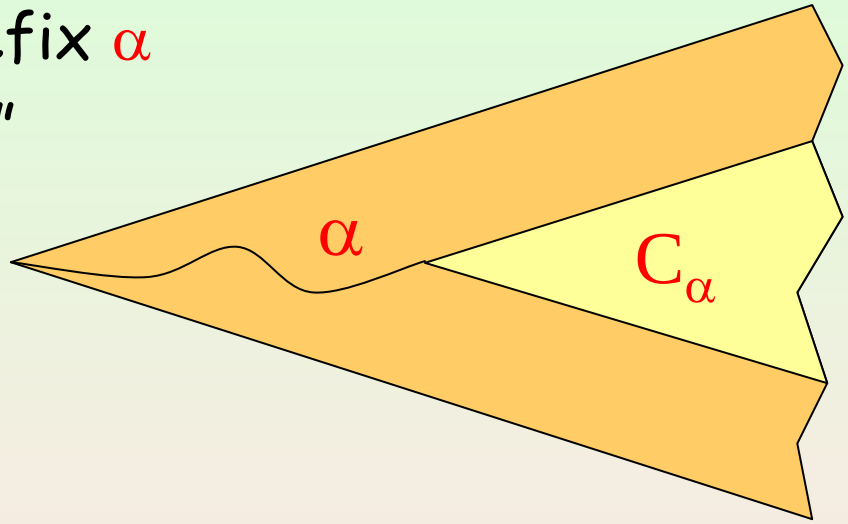
Example: Probabilistic Automata

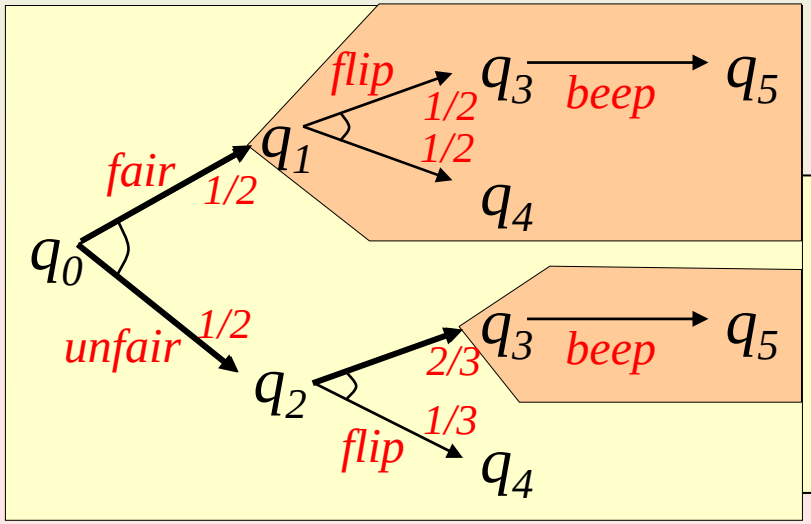


What is the probability of beeping?

Cones and Measures

- Cone of α
  - Set of executions with prefix α
  - Represent event "α occurs"
- Measure of a cone
  - Product edges of α



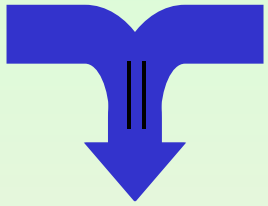


extends uniquely

σ-field generated by cones

Composition of Probabilistic Automata

$$A_1 = (Q_1, q_1, E_1, H_1, D_1) \qquad A_2 = (Q_2, q_2, E_2, H_2, D_2)$$



$$A_1 \| A_2 = (Q_1 \acute{} Q_2 , (q_1,q_2) , E_1 \cup E_2 , H_1 \cup H_2 , D)$$

$$D = \left\{ (q,a,(s_1,s_2)) \;\middle|\; \begin{array}{l} \text{if } a \in E_i \cup H_i \text{ then } (\pi_i(q) , a , s_i) \in D_i \\ \text{if } a \notin E_i \cup H_i \text{ then } s_i = \pi_i(q) \end{array} \quad i \in \{1,2\} \right\}$$

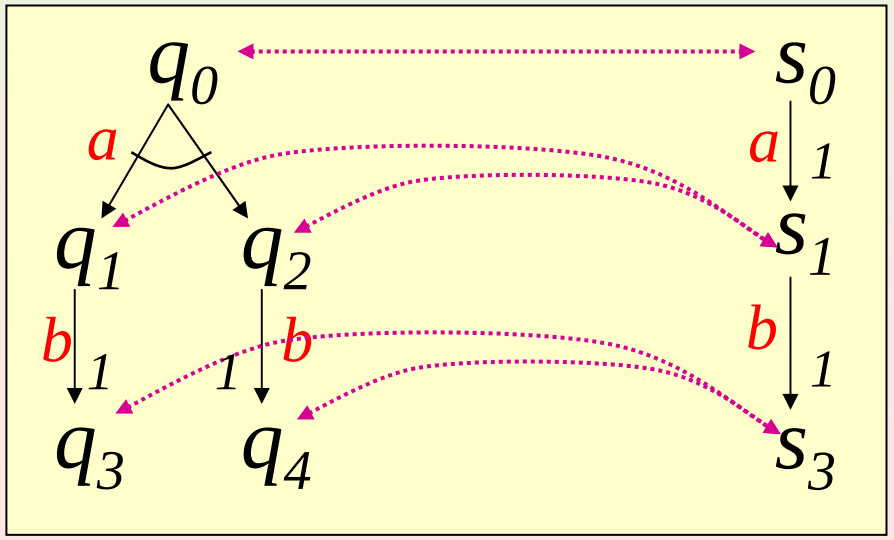
$$D = \left\{ (q,a,\mu_1 \times \mu_2) \;\middle|\; \begin{array}{l} \text{if } a \in E_i \cup H_i \text{ then } (\pi_i(q) , a , \mu_i) \in D_i \\ \text{if } a \notin E_i \cup H_i \text{ then } \mu_i = \delta(\pi_i(q)) \end{array} \quad i \in \{1,2\} \right\}$$

Strong Bisimulation on Probabilistic Automata

Strong bisimulation between A_1 and A_2

Relation $\boldsymbol{R} \subseteq Q \times Q$,

$Q = Q_1 \uplus Q_2$, such that



$\forall\, q, s, a, \mu \;\exists\, \mu'$

$q \xrightarrow{a} \mu$

$\boldsymbol{R}$ $\qquad$ $\boldsymbol{R}$

$s \xrightarrow{a} \mu'$

$\mu \;\boldsymbol{R}\; \mu'$ [LS89]

$\Leftrightarrow$

$\forall C \in Q/\boldsymbol{R} \,.\, \mu(C) = \mu'(C)$

Trace Distributions

The *trace* function is measurable

Trace distribution of μ

tdist(μ) : image measure under *trace* of μ

Trace distribution inclusion preorder

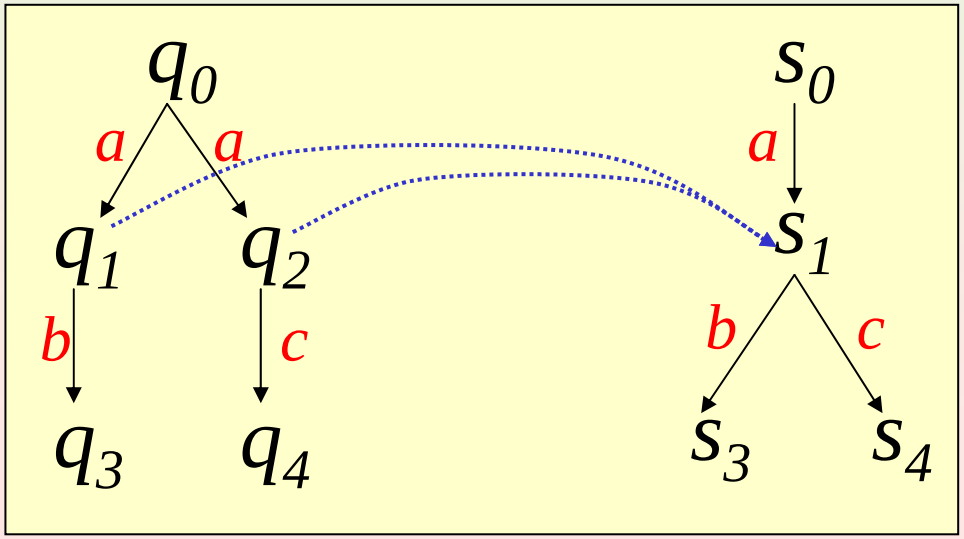
$A_1 \leq_{\mathrm{TD}} A_2$ iff $tdists(A_1) \subseteq tdists(A_2)$

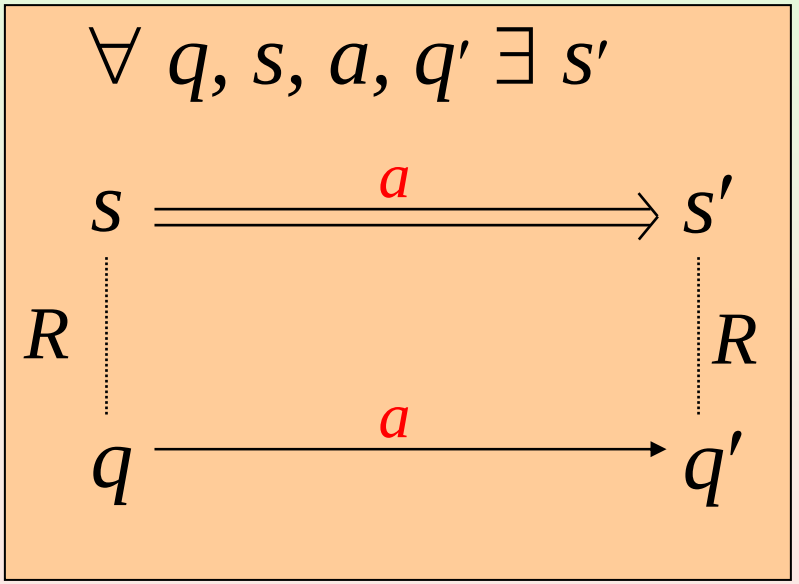
Trace Distribution Precongruence

- Coarsest precongruence included in preorder
- Simulations sound for precongruence
- There are also complete characterizations

Simulations (Automata)

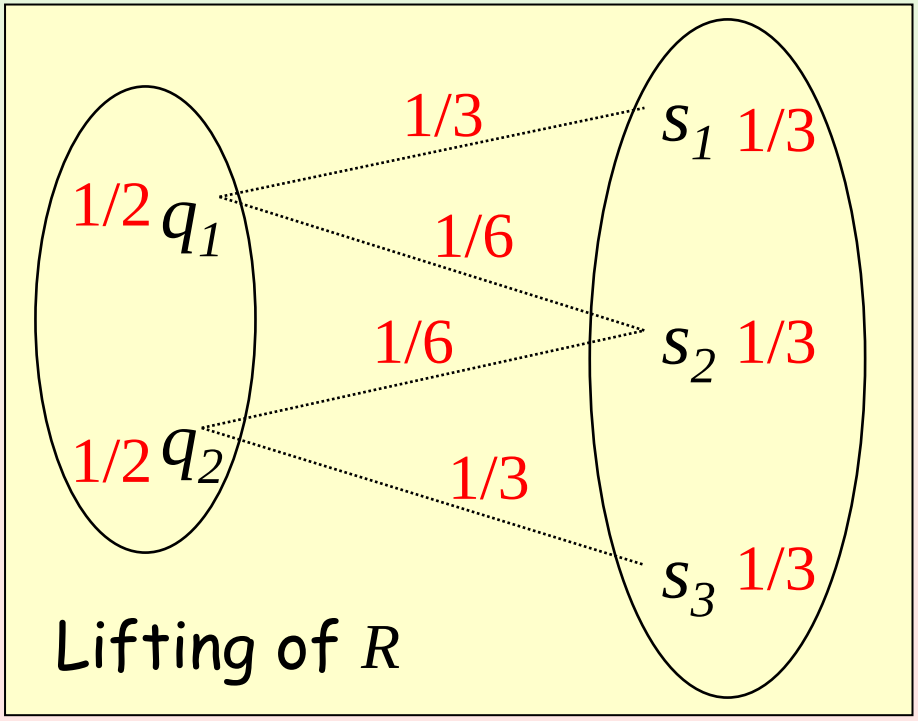
Forward simulation from A_1 to A_2 ($A_1 \leq_F A_2$)
Relation $R \subseteq Q_1 \times Q_2$ such that





Simulations on Probabilistic Automata

Simulation from A_1 to A_2 ($A_1 \leq_F A_2$)
Relation $R \subseteq Q_1 \times Q_2$ such that



$\forall\ q, s, a, \mu'\ \exists\ \sigma'$

$s \xRightarrow{a} \sigma'$

R $\quad$ R

$q \xrightarrow{a} \mu'$

The Consensus Problem

- There are n processes
- Each process proposes a value in {0,1}.
- Each process may decide on a value.
- Processes may fail by stopping.

Required properties

Validity: decide values that were proposed.

Agreement: no different decisions.

Wait-Free Termination: each non-failed process decides eventually.

Randomized Consensus

Validity: decide values that were proposed.

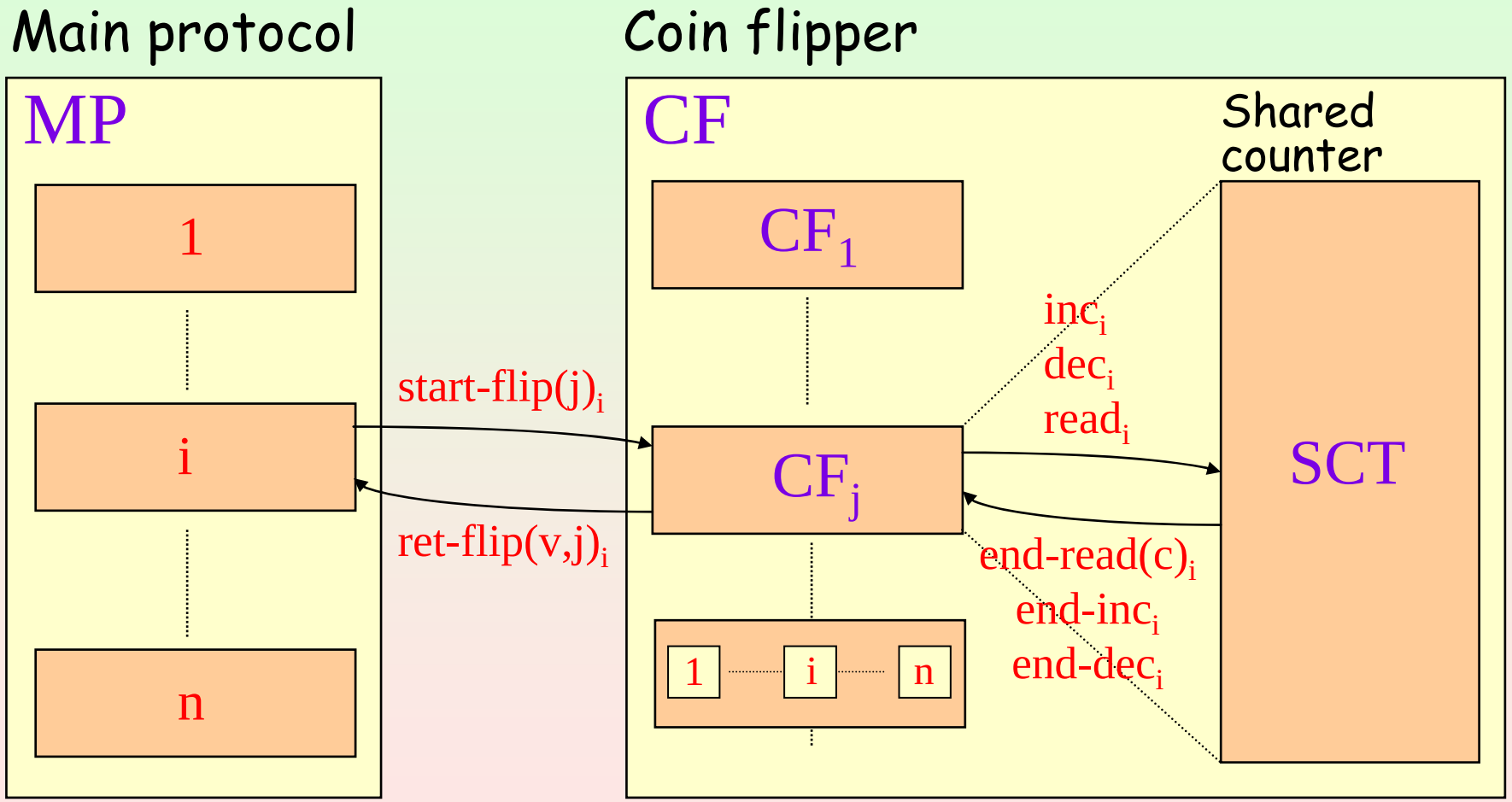
Agreement: no different decisions.

Probabilistic Wait-Free Termination:
each non-failed process decides with probability 1.

Aspnes and Herlihy

Termination within expected polynomial time.

Algorithm of Aspnes and Herlihy: Structure



Algorithm for the Main Protocol

```
scan pref and round
if obs-leader(i) and obs-agree(round(i)-1,v)
    then decide(v)
elseif obs-leaders-agree(v)
    then pref(i) = v and round(i) = round(i) + 1
    else pref(i) = ⊥
        scan pref and round
        if obs-leaders-agree(v)
            then pref(i) = v and round(i) = round(i) + 1
            else invoke coin flipper to choose a value for pref(i)
                 round(i) = round(i) + 1
```

Coding the Main Protocol

Read1(k)$_i$
Pre: pc_i = read1, $k \notin obs_i$
Eff: $values_i[k] \leftarrow pref(k)$
$rounds_i[k] \leftarrow round(k)$
$obs_i \leftarrow obs_i \cup \{k\}$
if $obs_i = \{1,\ldots,n\}$
then
$pc_i \leftarrow$ check1

Check2$_i$
Pre: pc_i = check2
Eff: if $\exists_{v \in \{0,1\}}$ obs-leader-agree(v)
then
pref(i) $\leftarrow$ obs-leader-value
round(i) $\leftarrow$ $rounds_i[i]+1$
$obs_i \leftarrow \emptyset$
$pc_i \leftarrow$ read1
else
$pc_i \leftarrow$ flip

Proof of Correctness: Safety

Validity: ordinary invariant proof

agree(1,v) and obs-agree(1,v)

Agreement: ordinary invariant proof

(obs-agree(r-1,v) obs-leader(i)$_i$ obs$_i$ = {1,…,n})

implies

$\Rightarrow$ agree(r,v)

Proof of Correctness: Progress

Assume the invocations to the coin flippers on non-failing ports always get an answer (M1)

$$R \xrightarrow{1} F_0 \cup F_1 \cup D$$

Assume that the all answers at round r are 0 (where, for $s \in F_0$, max-round(s) = r) (M2)

$$F_0 \xrightarrow{2} D$$

Assumptions on Coin Flippers

Each coin flipper satisfies the properties

C1: each invocation on a non-failing port gets an answer with probability 1.

C2: fixed a value v in {0,1} the probability that all the answers are v is at least p.

We will show that p is independent of n.

Combination of Claims

From *C1* and *C2* and *M1* and *M2*

$$R \xrightarrow{1}_{1} F_0 \cup F_1 \cup D$$
$$F_0 \xrightarrow{2}_{p} D$$

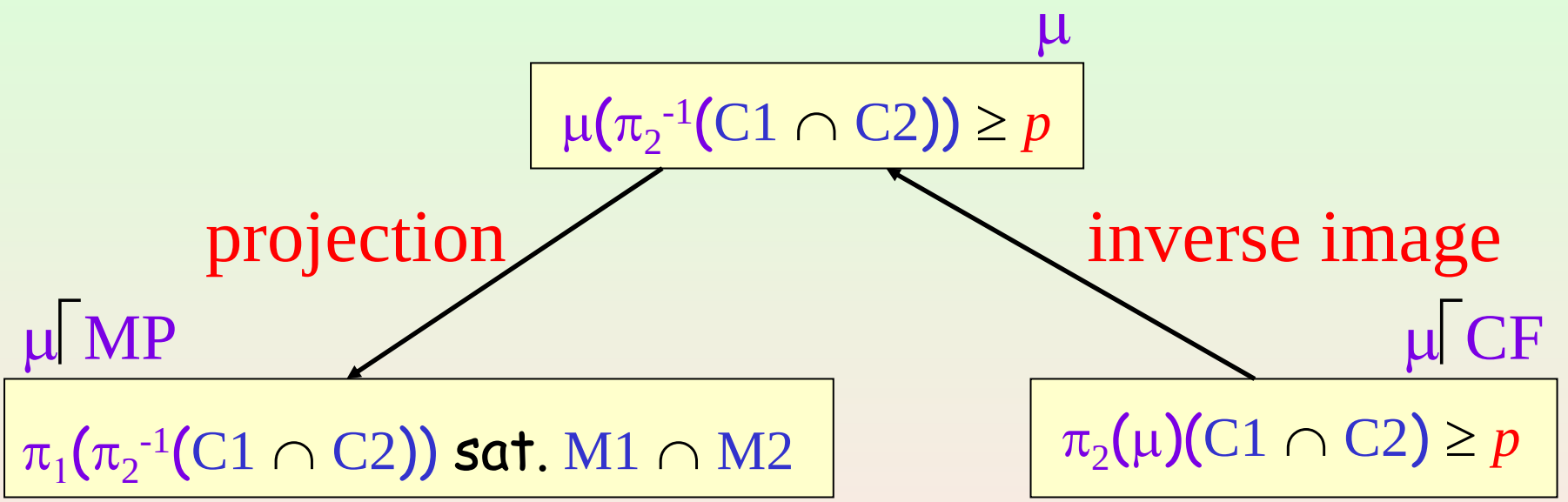
Combining the statements above

$$R \xrightarrow{3}_{p} D$$

Thus termination within expected 3/p rounds.

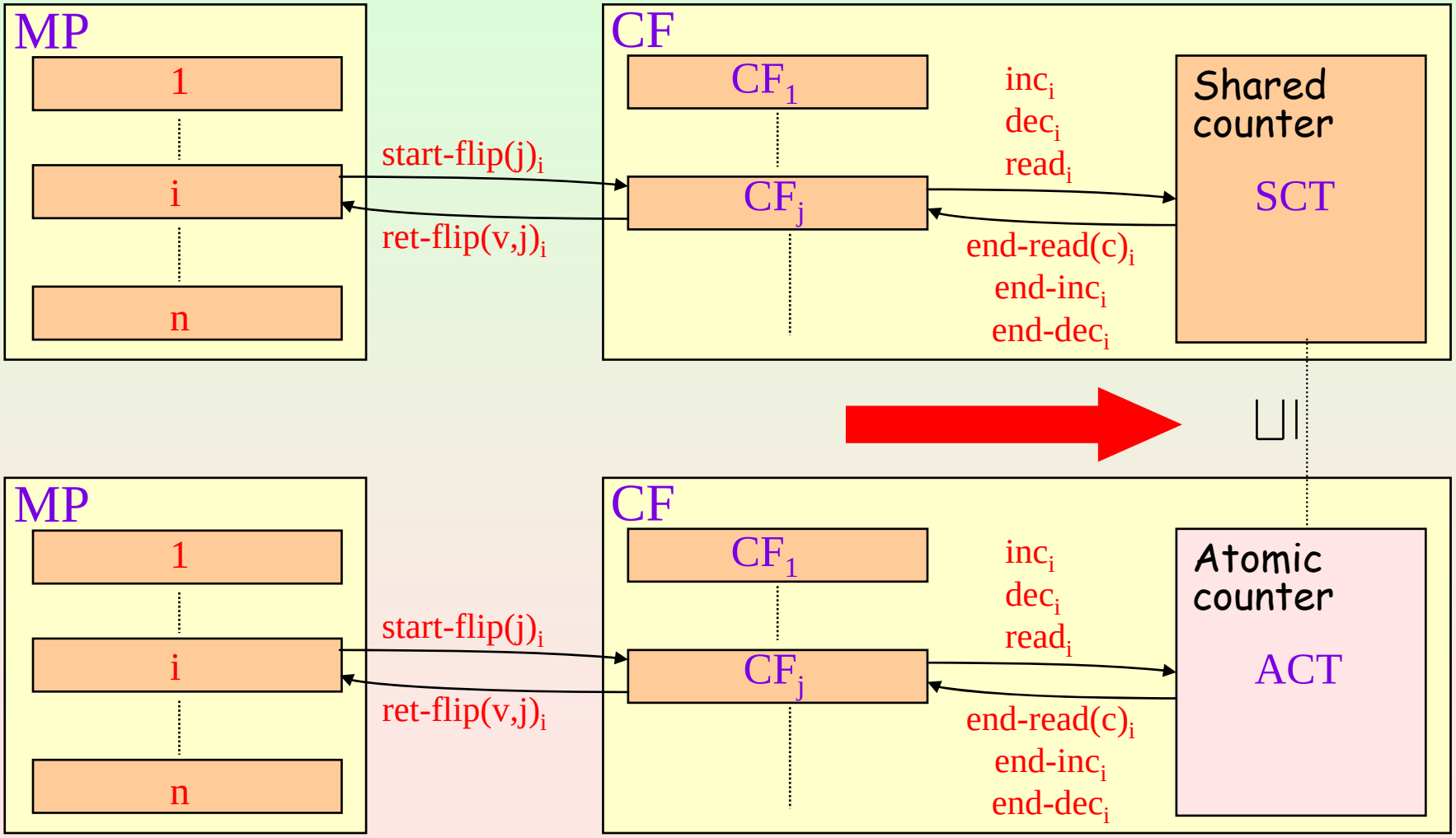
Formal Argument

Let μ start in a state s of F_0.

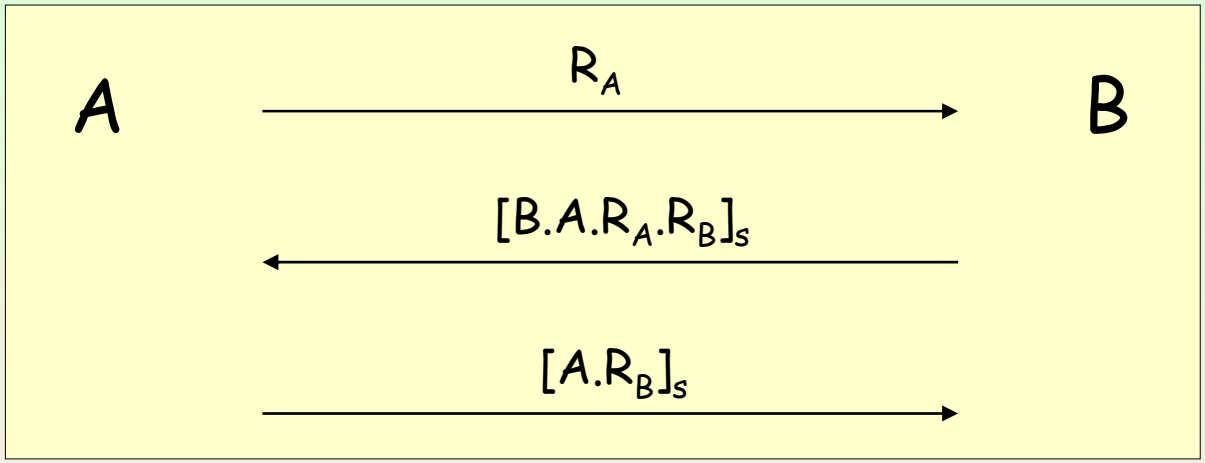


$$F_0 \xrightarrow{2}_{p} D$$

The Global Picture



Bellare and Rogaway MAP1 Protocol



- Nonces are generated randomly
- The key s is the secret for a Message Authentication Code
  - Specifically, MAC based on pseudo-random functions

Nonces

- Number ONCE
  - Typically drawn randomly
- Claim
  - For each constant c and polynomial p
  - There exists $\boldsymbol{k}$ such that for each $k \geq \boldsymbol{k}$
  - If $n_1, n_2, \ldots, n_{p(k)}$ are random nonces from $\{0,1\}^k$
  - Then $\Pr[\exists_{i \neq j}\, n_i = n_j] < k^{-c}$

Message Authentication Code

- Triple (G,A,V)
  - G on input 1^k generates $s \in \{0,1\}^k$
  - For each s and each a
    - Pr[V(s,a,A(s,a))=1]=1
- Forger
  - On input 1^k obtains MAC of strings of its choice
  - Outputs a pair (a,b)
  - Successful if V(s,a,b)=1 and a different from previous queries
- Secure MAC
  - Every feasible forger succeeds with negligible probability

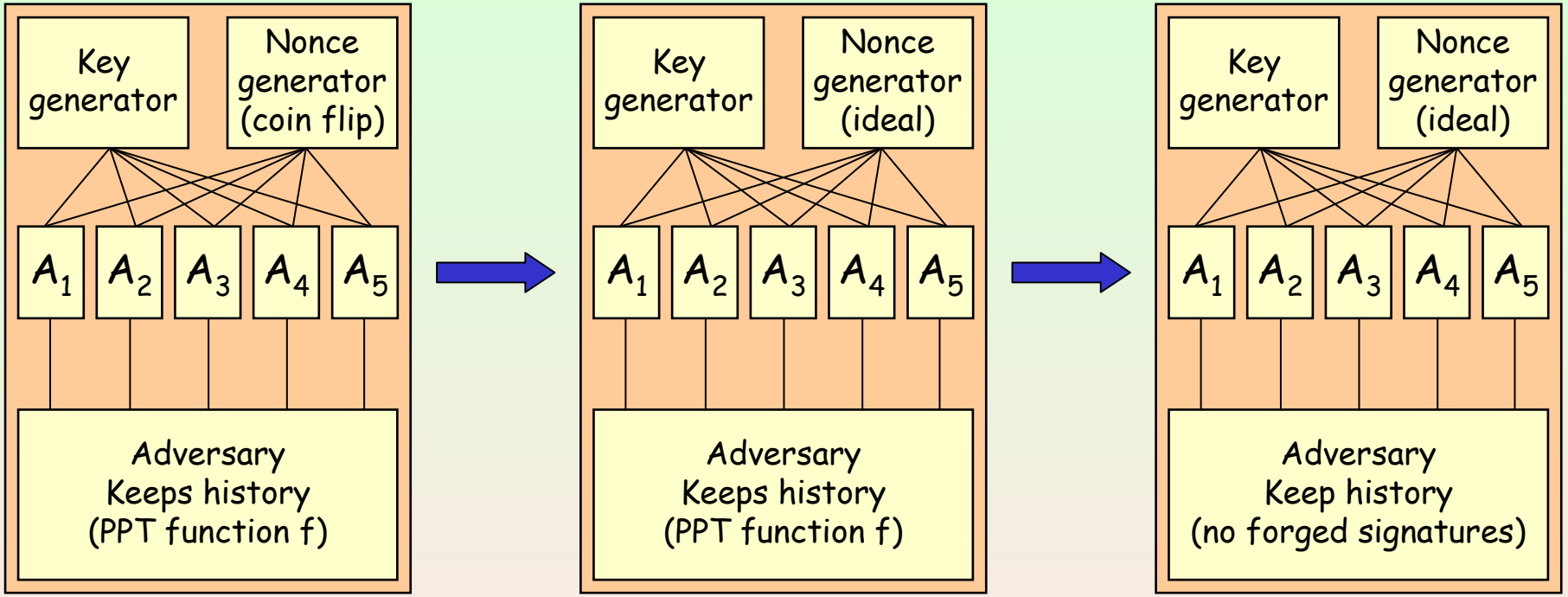
MAP1: Matching Conversations

- Matching conversation between A and B
  - Every message from A to B delivered unchanged
    - Possibly last message lost
    - Response from B returned to A
  - Every message received by A generated by B
    - Messages generated by B delivered to A
    - Possibly last message lost
- Correctness condition
  - Matching conversation implies acceptance
  - Negligible probability of acceptance without matching conversation

MAP1: Correctness Proof

- Let A be a PPT machine that interacts with the agents
- Show that A induces "no-match" with negligible probability
  - Argue that repeated nonces occur with negligible probability
  - Argue that A is an attack against a message authentication code
- Features
  - Relies on underlying pseudo-random functions
  - Proves correctness assuming truly random functions
  - Builds a distinguisher for PRFs if an attack exists
- Criticism
  - The arguments are semi-formal and not immediate
  - Three different concepts intermixed
    - Nonces
    - Message authentication codes
    - Matching conversations

MAP1: Hierarchical Analysis



- Agents indexed by X, Y, t
- Need to find suitable simulations
  - Step conditions lead to local arguments
  - Yet transitions cannot be matched exactly

Nonce Generators

- State
  - $value_{X,Y,t}$ initially $\perp$
  - $FreshNonces$ initially $\{0,1\}^k$

- Transitions
  - Input $NonceRequest_{X,Y,t}$
  - Effect
    - Let $v \in_R \{0,1\}^k$
    - $value_{X,Y,t} = v$
    - $FreshNonces = FreshNonces - \{v\}$
  - Output $NonceResponse_{X,Y,t}(n)$
  - Precondition
    - $n = value_{X,Y,t}$
  - Effect
    - $value_{X,Y,t} = \perp$

Coin flip

Ideal

- Let $v \in_R FreshNonces$

Adversary

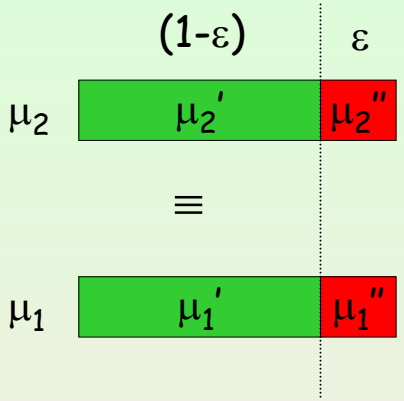
- Keeps a variable *history*
  - Holds all previous messages
- Real adversary
  - Runs a cycle where
    - Computes the next message to send using a PPT function f
    - Sends the message
    - Waits for the answer if expected
- Ideal adversary
  - Highly nondeterministic
  - Stores all input
  - Sends messages that do not contain forged authentications

Problems with Simulations

- Problem
  - Consider a transition of the real nonce generator
  - With some probability there is a repeated nonce
  - The ideal nonce generator does not repeat nonces
  - Thus, we cannot match the step
- Solution
  - Match transitions up to some error

Approximate Simulations [ST07]

- Change equivalence on measures
  - $\mu_1 \equiv_\varepsilon \mu_2$ iff
    - $\mu_1 = (1-\varepsilon)\mu_1' + \varepsilon\mu_1''$
    - $\mu_2 = (1-\varepsilon)\mu_2' + \varepsilon\mu_2''$
    - $\mu_1' \equiv \mu_2'$



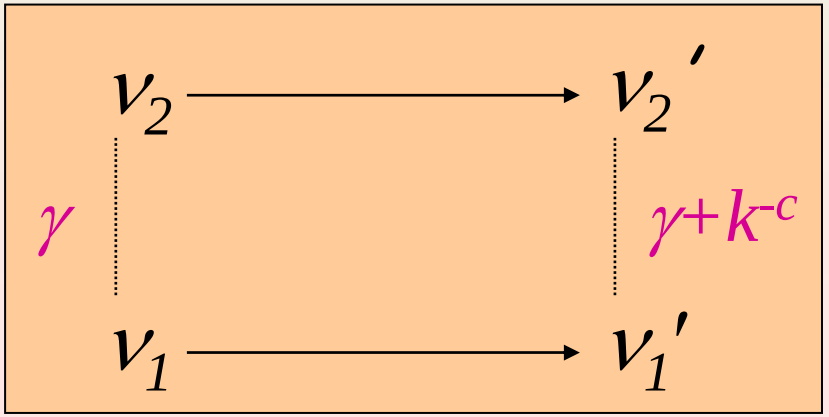
- Add parameterizations
  - Consider families of PIOA parameterized by k
- Require ε smaller than any polynomial in k

- Absence of simulation becomes failure
  - Existence of a forger for a signature
  - Random nonces that are equal
  - Existence of a distinguisher

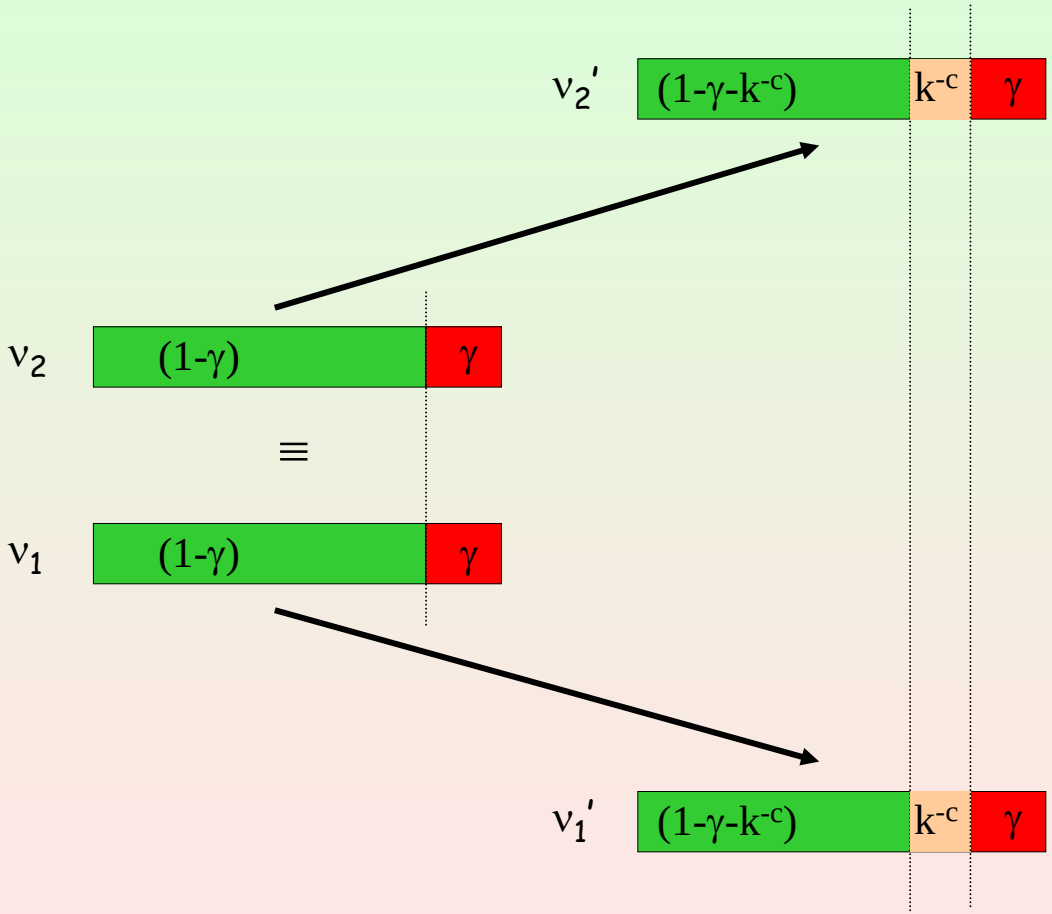
Approximate Simulations

$\{A_k\}$ $\{R_k\}$ $\{B_k\}$

- For each constant c and polynomial p
- There exists $\boldsymbol{k}$ such that for each $k \geq \boldsymbol{k}$
- Whenever
  - ν_1 reached within $p(k)$ steps in A_k
  - $\nu_1\ L(R_k,\gamma)\ \nu_2$
  - $\nu_1 \rightarrow \nu_1'$
- There exists ν_2' such that
  - $\nu_2 \rightarrow \nu_2'$
  - $\nu_1'\ L(R_k,\gamma+k^{-c})\ \nu_2'$



Approximate Simulations
Step Condition



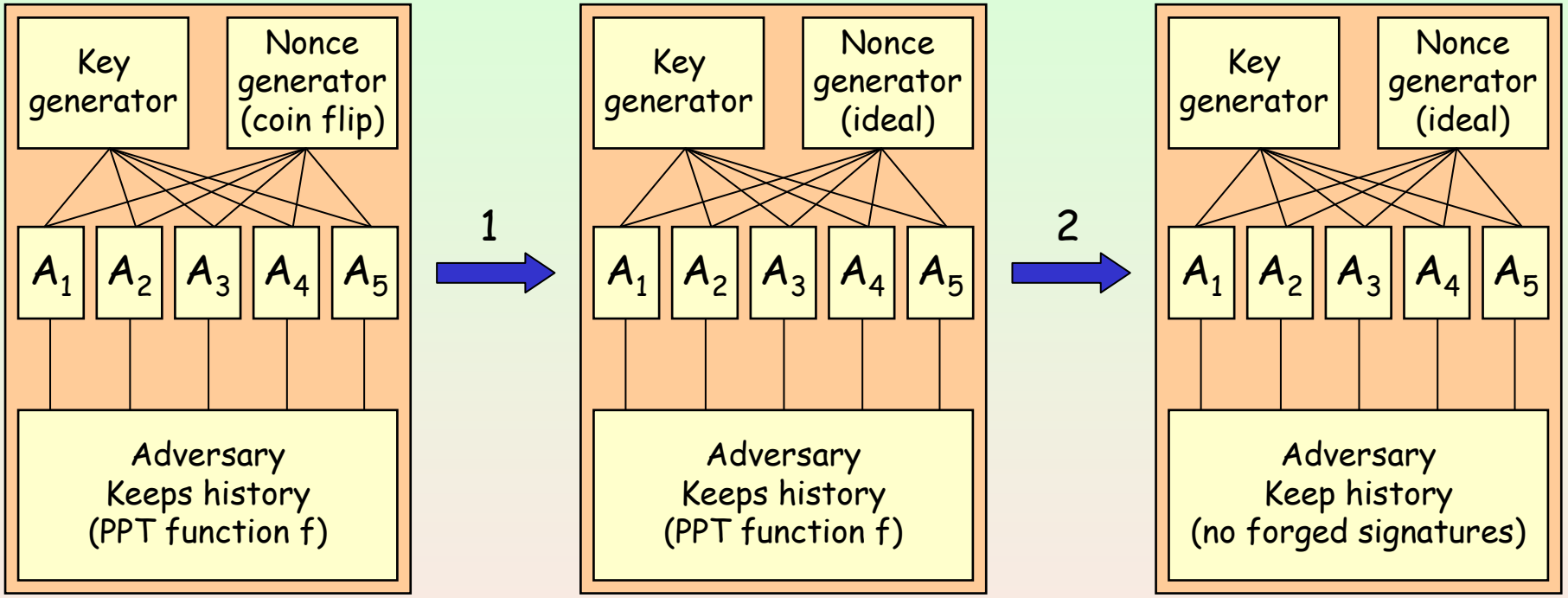
Execution Correspondence under Approximated Simulations

If $\{A_k\}$ $\{R_k\}$ $\{B_k\}$ then

- For each constant c and polynomial p
- There exists $\boldsymbol{k}$ such that for each $k \geq \boldsymbol{k}$
- For each scheduler σ_1
  - ν_1 reached within $p(k)$ steps in A_k with σ_1
- There exists σ_2 such that
  - ν_2 reached within $p(k)$ steps in B_k with σ_2
  - ν_1 $L(R_k, p(k)k^{-c})$ ν_2

- Observation
  - $p(k)k^{-c}$ can be smaller than any $k^{-c'}$ by choosing $c=c'+degree(p)$

Example: Approximate Simulations Bellare-Rogaway MAP1 Protocol



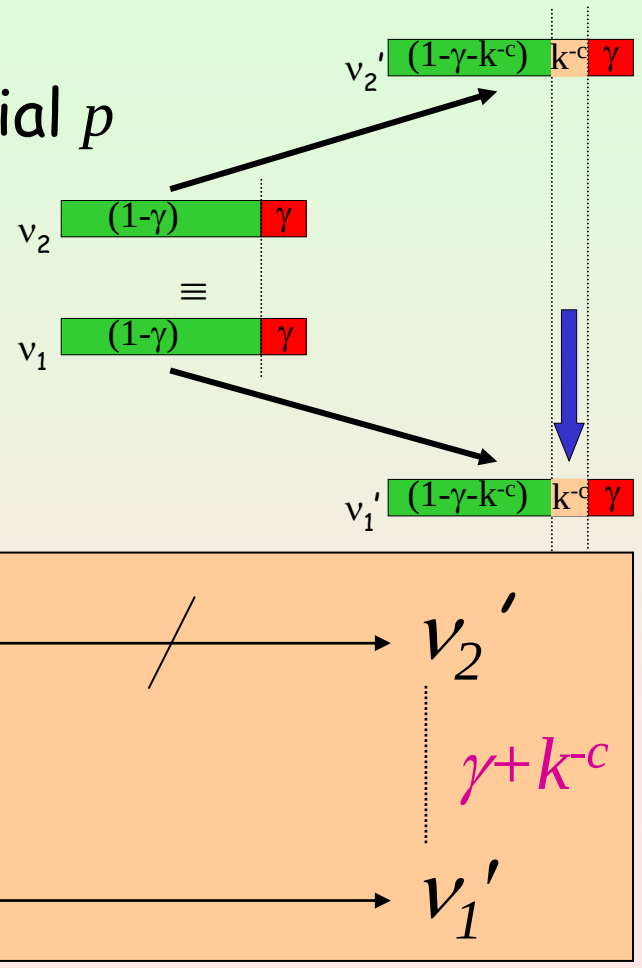
- Negation of the step condition
  - 1: Two random nonces are equal with high probability
  - 2: Function f defines a forger for a signature scheme

Negation of Step Condition Nonce Generation

$\{A_k\}$ $\{R_k\}$ $\{B_k\}$

- There exists constant c and polynomial p
- For each $\boldsymbol{k}$ there exists $k \geq \boldsymbol{k}$
- There exists
  - ν_1 reached within $p(k)$ steps in A_k
  - $\nu_1\ L(R_k,\gamma)\ \nu_2$
  - $\nu_1 \rightarrow \nu_1'$
- There is no ν_2' such that
  - $\nu_2 \rightarrow \nu_2'$
  - $\nu_1'\ L(R_k,\gamma+k^{-c})\ \nu_2'$

- Since ν_1 is reached in ν_1'
  - Probability at least k^{-c}



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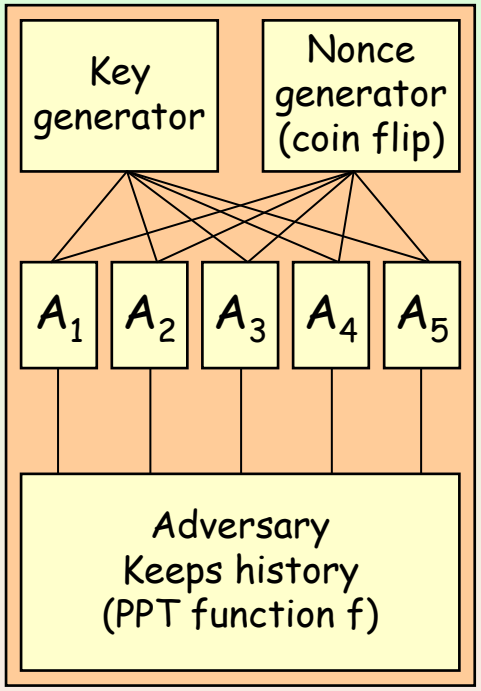
Applicability

- Dolev-Yao Model
  - Soundness w.r.t. indistinguishability
  - How about correspondence of computations?
- Cryptographic library
  - More rigorous/local proofs?
  - Alternative to error sets?
- Game transformations
  - Proof method?

Problems with Nondeterminism
MAP1 Protocol [BR93]



- Authentication protocol
  - Symmetric key signature schema
  - Computational Dolev-Yao
  - Adversary queries agents
- Potential problems
  - Let s be the shared key
  - Adversary queries k agents
  - Agent i replies if i^{th} bit of s is 1
  - The adversary knows the shared key
- Solution
  - One query at a time
  - Wait for the answer (agents as oracles)

Approaches to Nondeterminism

- Reduce the power of the scheduler
  - Process Algebras
    - Scheduler sees only enabled action type
  - UC framework
    - ITMs have a token passing mechanism
    - No nondeterminism
  - Reactive simulatability
    - Again token passing mechanism
    - Nondeterminism based on local information only
  - Task PIOAs
    - Define equivalence classes of states and actions
    - Scheduler sees only equivalence classes, not elements
  - Careful specifications
    - Avoid dangerous nondeteminism in the specification
    - Is it always possible?

Concluding Remarks

- Formal methods are useful
  - Semi-formal proofs may be wrong
  - Semi-formal proofs require attention
- There are several approaches
  - Computational, symbolic, compositional
  - Suitable for crypto-primitives or protocols
- We need proof techniques
  - Algebraic, symbolic, automata theoretic
- Nondeterminism arises and gives problems
  - Restrict resolution of nondwterminism
  - Avoid dangerous nondeterminism

What Else?

- A lot to understand on approximated simulations
  - Are they connected to metrics?
  - Can we define them incrementally
    - How far can we go without polynomial bounds?
  - How about approximated language inclusion?

- Need more techniques
  - Can we have a uniform view?
  - Can we relate better computational and symbolic approaches?
  - Any crucial differences between crypto-primitives and protocols?
  - How about cross migration of techniques?

- Need more automation
  - … but we need to understand what we automate