Formal Methods for Security
Why? – How?

Roberto Segala
University of Verona

Formal Methods and Formal Verification

Why?

Why Formal Analysis?

- 1994: The pentium processor computes wrong divisions
  - INTEL forced to replace most processors
  - Economic damage of 450 million US Dollars

- 1995: The software MacInTax spreads the secrets of US tax payers
  - Error in the debug code distributed with MacInTax
  - Users can use it to access the server of Intuit
  - Everybody can read and modify any tax form

Why Formal Analysis?

- 1995: Problems in Denver Airport
  - The fully automated baggage system fails
  - Scheduled to open in 1993
  - The system looses or tears apart luggage
  - Considerable congestion
  - Considerable lack of design
  - In 2005 the system is still not working
  - The system is too complex
  - Extensive research activity is necessary

Why Formal Analysis?

- 1996: Vector Ariane 5 explodes during take-off
  - The control software assigns a 64 bit number to a 16 bit variable
  - The code was recycled from Ariane 4
  - Ariane 5 is fast and its lateral speed does not fit in 16 bits
  - Result: overflow – the system shuts down
  - The back up computer is started
  - … but the software is the same
  - Result: again overflow – the system shuts down
  - Ariane, without guidance, self destroyes
  - Damage: 1 billion Euros

Why Formal Analysis?

- 1982 Mutual exclusion solved with small shared variables
  - Rabin proposes a randomized distributed algorithm
  - The proof is semi-formal but credible
- 1990 Some problems appear
  - Nancy Lynch gives a lecture on Rabin's algorithm
  - Roberto Segala is the scribe and tries to formalize the proof
  - Problem in an informally obvious step
    - Two events are compared but they belong to different probability spaces
  - Nondeterminsm is the cause of the problem
- 1991 An attack is found
- Later many other algorithms turned out to be bogus

Why Formal Analysis?

- 1978: Needham and Schroeder
  - Propose an authentication protocol
  - The correctness proof is semi-formal
- 1981: Problems with freshness
  - Replay attacks are possible
- 1995: An attack found
  - Parallel sessions may lead to attack
- Needham: you changed my definitions
- Later: many protocols have been attacked

Lessons that we can Learn

- Formal methods are useful (necessary)
  - Need to define what we want
    - Objectives should be clear and accepted
    - We should communicate with others
  - Need to prove properties rigorously
    - We may miss pieces otherwise
    - We need techniques
  - Need modular verification techniques
    - We want to reuse existing proofs
  - Need ways to automate the analysis
    - Large systems require considerable effort

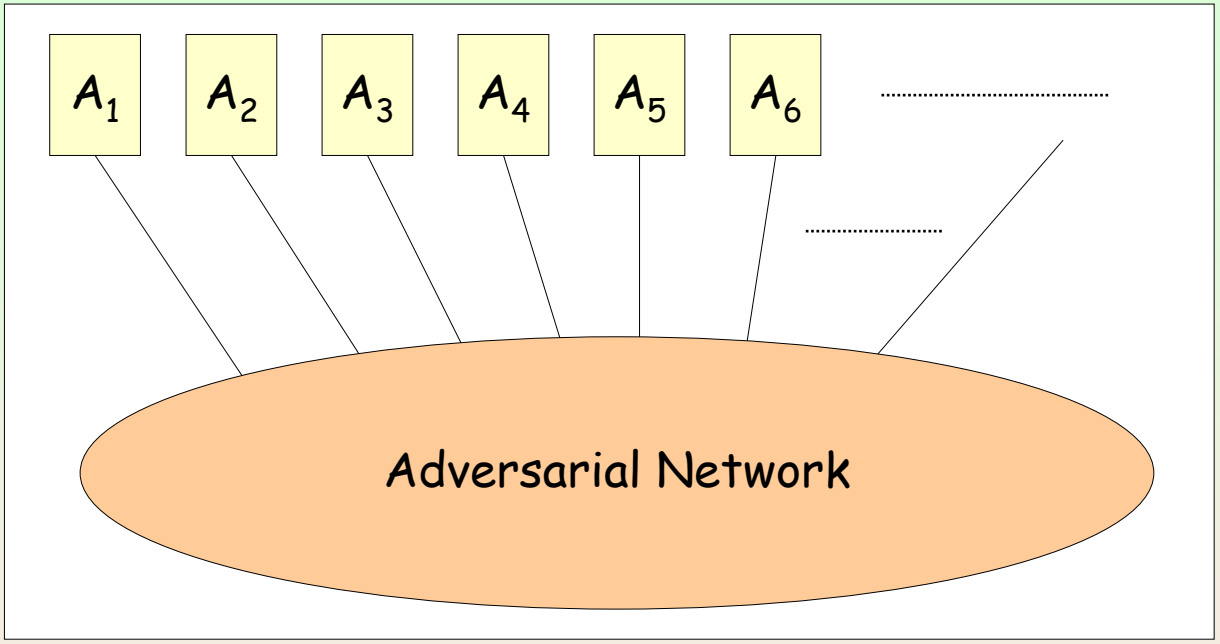
Formal Methods for Security: How?

- Provable security [GM84]
  - Based on Turing Machines (computational model)
  - Proofs by reduction to known difficult problems
- Dolev-Yao model [DY83]
  - Based on automata theory
  - Perfect cryptography
  - Need to know relationship to computational model
- Universally composable security [Can01]
  - Based on Interactive Turing Machines
  - Specification includes accepted attacks
- Reactive Simulatability [PW01]
  - Based on Probabilistic I/O Automata
  - Similar to UC framework

Provable Security

- Let h be a computationally hard function
- Let C be a cryptographic primitive
  - Collection of PPT algorithms that compute some functions
- State correctness of C as follows
  - There is no PPT algorithm A that computes some function f
- Prove correctness of C as follows
  - Suppose for the sake of contradiction that A exists
  - Build a PPT algorithm for h that uses A as a black box
  - This contradicts the hardness of h
- Correctness of C relies on hardness of h

Dolev-Yao Model



- Agents communicate through adversarial network
  - Network remembers everything
  - Network may block or reroute messages
  - Network may cast new messages

Dolev-Yao Model: Assumptions

- Computational
  - Messages are bit strings
  - Adversary governed by PPT functions
- Symbolic (typical use of the model)
  - Messages are symbols
  - Cryptography is perfect
  - Adversary power limited by a deduction system
    - Nonces are always fresh
    - No ability to decrypt without decryption key
  - Adversary is nondeterministic

Symbolic Dolev-Yao Model

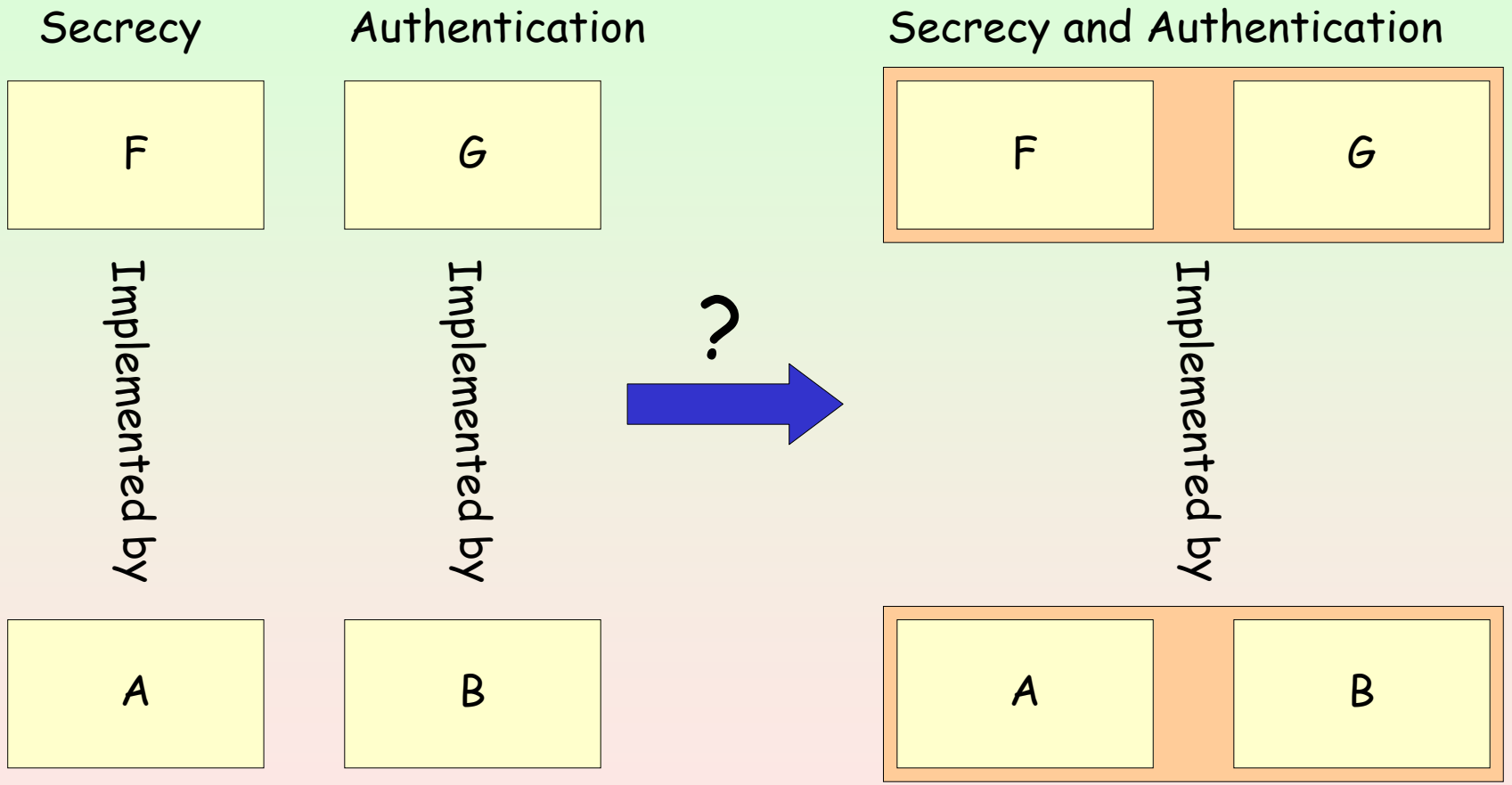
- Analysis is simple
  - The system is described by an automaton
  - Show that no path leads to failure or attack
  - Plenty of techniques from concurrency theory
    - Invariants
    - Compositional analysis
    - Language properties
    - Model checking
- Sound with respect to computational [AR00]
  - Attack in computational model yields attack in symbolic model
  - Need some assumptions on underlying cryptoprimitives
    - Non malleability

Symbolic Dolev-Yao Deductions

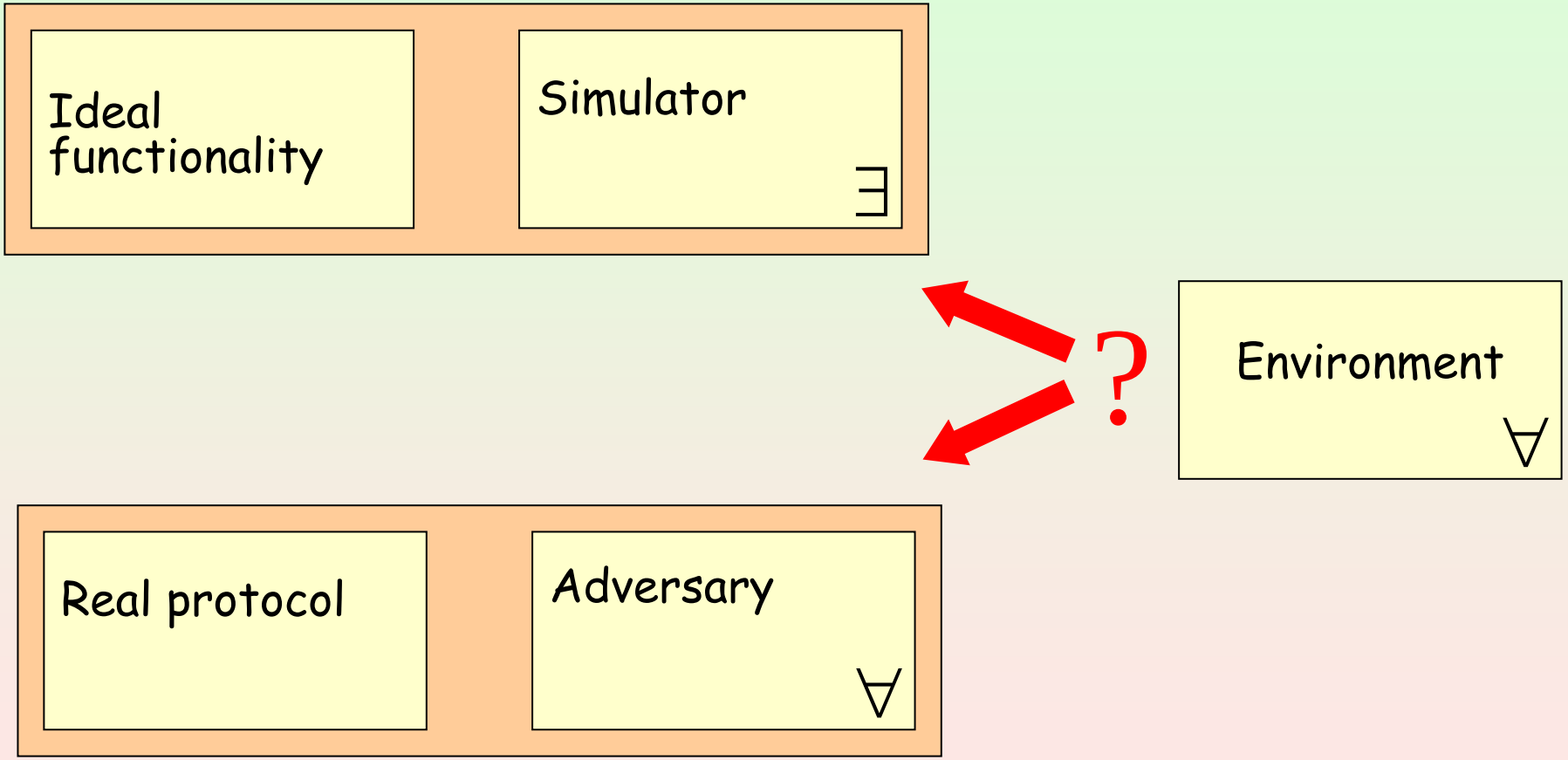
- $A \mid\!- X, \quad A \mid\!- Y \quad \Rightarrow A \mid\!-(X,Y)$
- $A \mid\!- (X,Y) \quad \Rightarrow A \mid\!- X$
- $A \mid\!- (X,Y) \quad \Rightarrow A \mid\!- Y$
- $A \mid\!- X, \quad A \mid\!- k \quad \Rightarrow A \mid\!- \{X\}_k$
- $A \mid\!- \{X\}_k, \quad A \mid\!- k \quad \Rightarrow A \mid\!- X$

- Automaton transitions
  - Agents add messages to adversary
  - Adversary casts messages according to deductions
- Invariants
  - Signature deducible only if it exists already
- Property
  - Answers always generated by correct agents

UC [Can01] and RSim [PW01] Motivation



UC-Framework [Canetti]



Reactive Simulatability [Pfitzmann Waidner]

- Similar to UC Framework
- Based on PIOAs rather than ITMs
- More elaborated on verification techniques
- Large collection of definitions
  - Crypto library [BPW03]

Fine, but how do we prove Facts?

- Provable security
  - Semi-formal arguments
  - A lot of wording
- Dolev-Yao
  - Semi-formal arguments
  - … or typical arguments from concurrency theory
- UC Framework
  - Semi-formal arguments
- Reactive simulatability
  - Semi-formal arguments
  - "Simulation" up to "error sets"
  - Negligible probability of error sets

Can we be More Rigorous?

- Use Dolev-Yao and Soundness
  - Concurrency theory has plenty of techniques
- Use Process Algebraic formalisms [MRST06 and earlier]
  - Expressions denote PPT computable functions
  - Equivalence denotes undistinguishability
  - Axiomatic reasoning
- Use game transformations [Sho04,Bla05]
  - Correctness in provable security expressed as a game
  - Transform games preserving correctness
- Use Automata Theory [CCKLLPS06,ST07]
  - Add computational assumptions
  - Extend known techniques (simulation method)

Game Transformations
Example: El Gamal

- $x \leftarrow Z_q$, $\alpha \leftarrow \gamma^x$
- $r \leftarrow R$, $(m_0,m_1) \leftarrow A(r,\alpha)$
- $b \leftarrow \{0,1\}$, $y \leftarrow Z_q$, $\beta \leftarrow \gamma^y$, $\delta \leftarrow \alpha^y$, $\xi \leftarrow \delta m_b$
- $\boldsymbol{b} \leftarrow A(r,\alpha,\beta,\xi)$

$\gamma^x, \gamma^y, \gamma^{xy}$

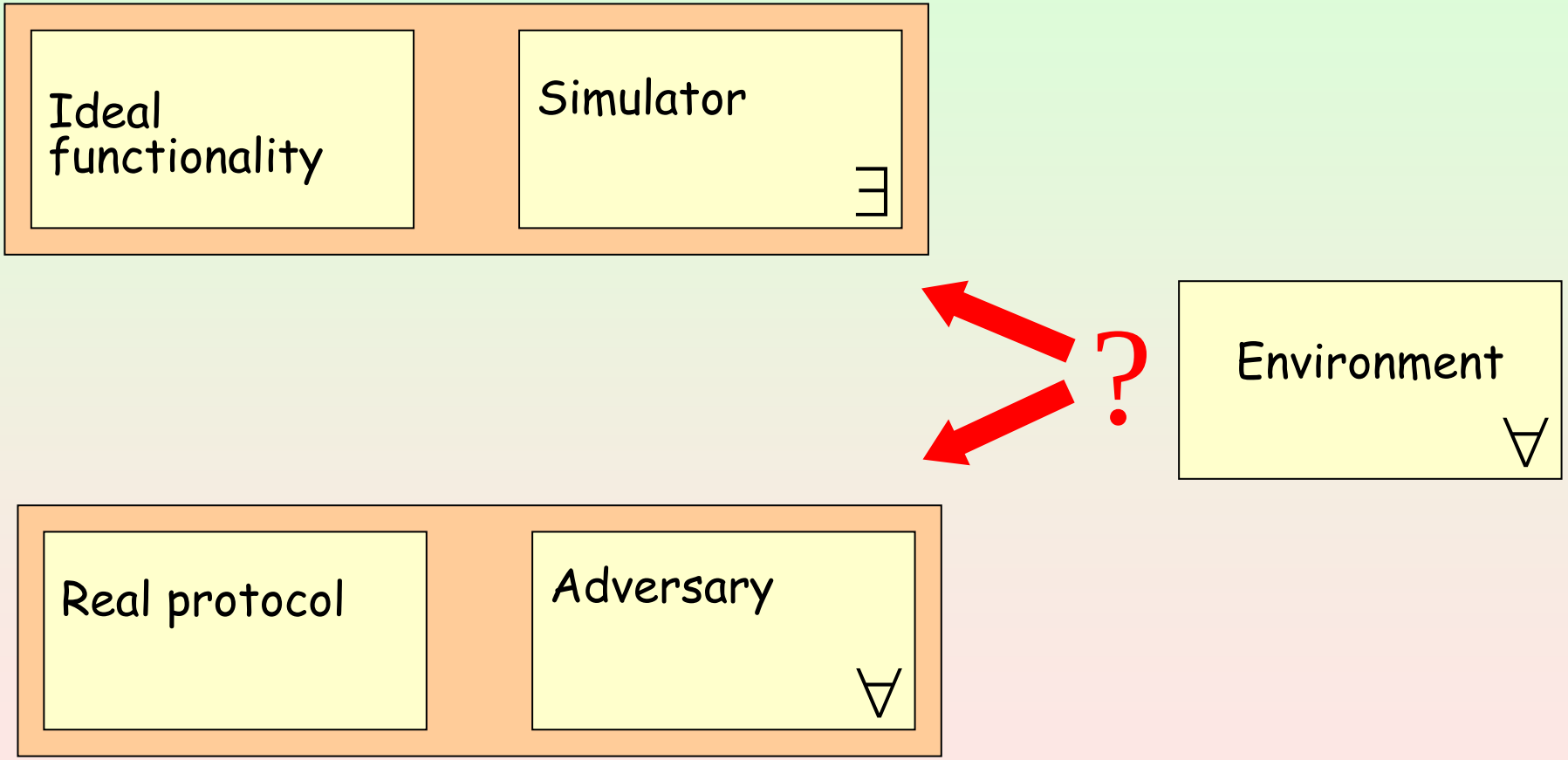
$D(\alpha,\beta,\delta)$

- $x \leftarrow Z_q$, $\alpha \leftarrow \gamma^x$
- $r \leftarrow R$, $(m_0,m_1) \leftarrow A(r,\alpha)$
- $b \leftarrow \{0,1\}$, $\xi \leftarrow \delta m_b$
- $\boldsymbol{b} \leftarrow A(r,\alpha,\beta,\xi)$

- $x \leftarrow Z_q$, $\alpha \leftarrow \gamma^x$
- $r \leftarrow R$, $(m_0,m_1) \leftarrow A(r,\alpha)$
- $b \leftarrow \{0,1\}$, $y \leftarrow Z_q$, $\beta \leftarrow \gamma^y$, $z \leftarrow Z_q$, $\delta \leftarrow \alpha^z$, $\xi \leftarrow \delta m_b$
- $\boldsymbol{b} \leftarrow A(r,\alpha,\beta,\xi)$

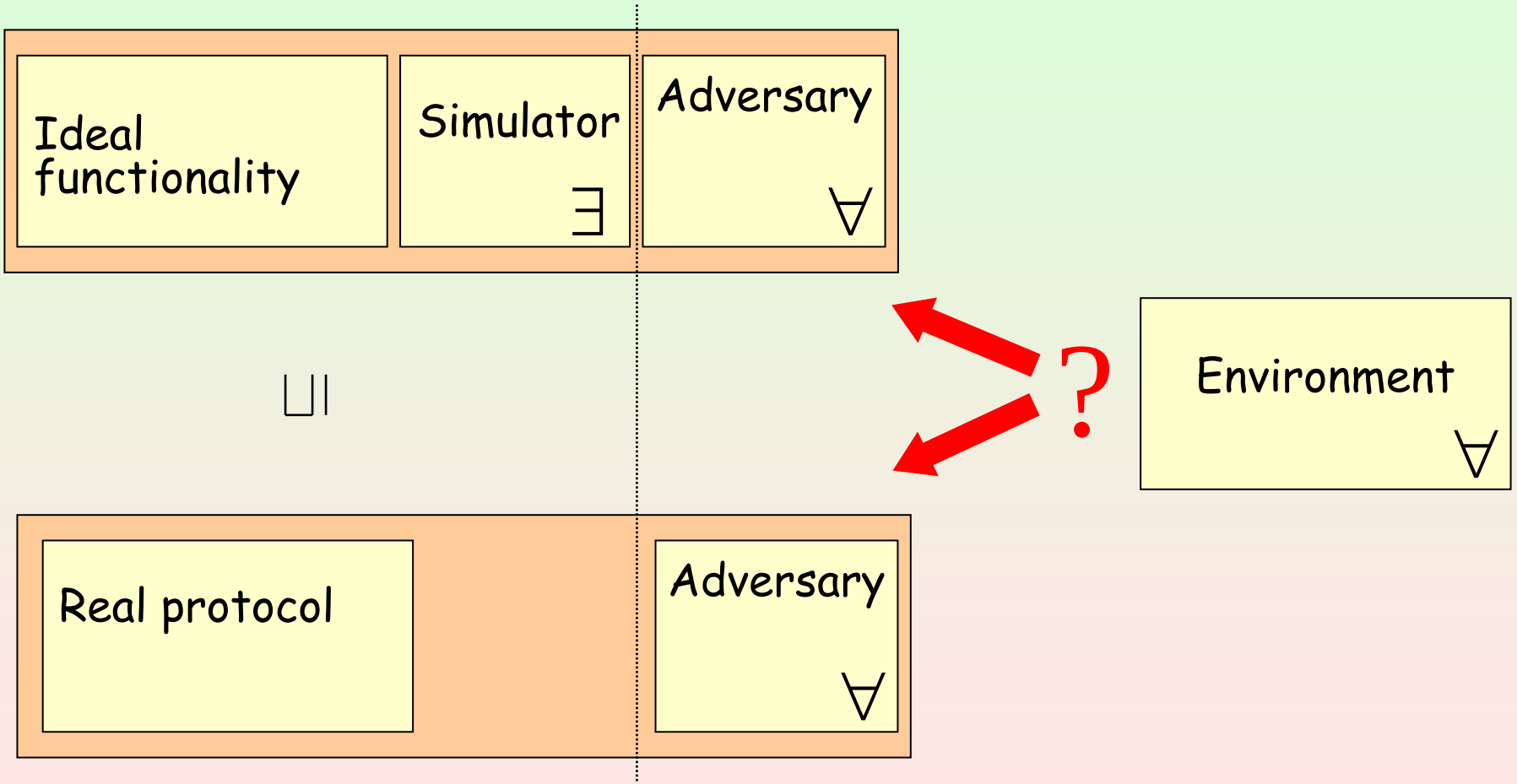
$\gamma^x, \gamma^y, \gamma^w$

UC-Framework [Canetti]



UC-Security with PIOAs

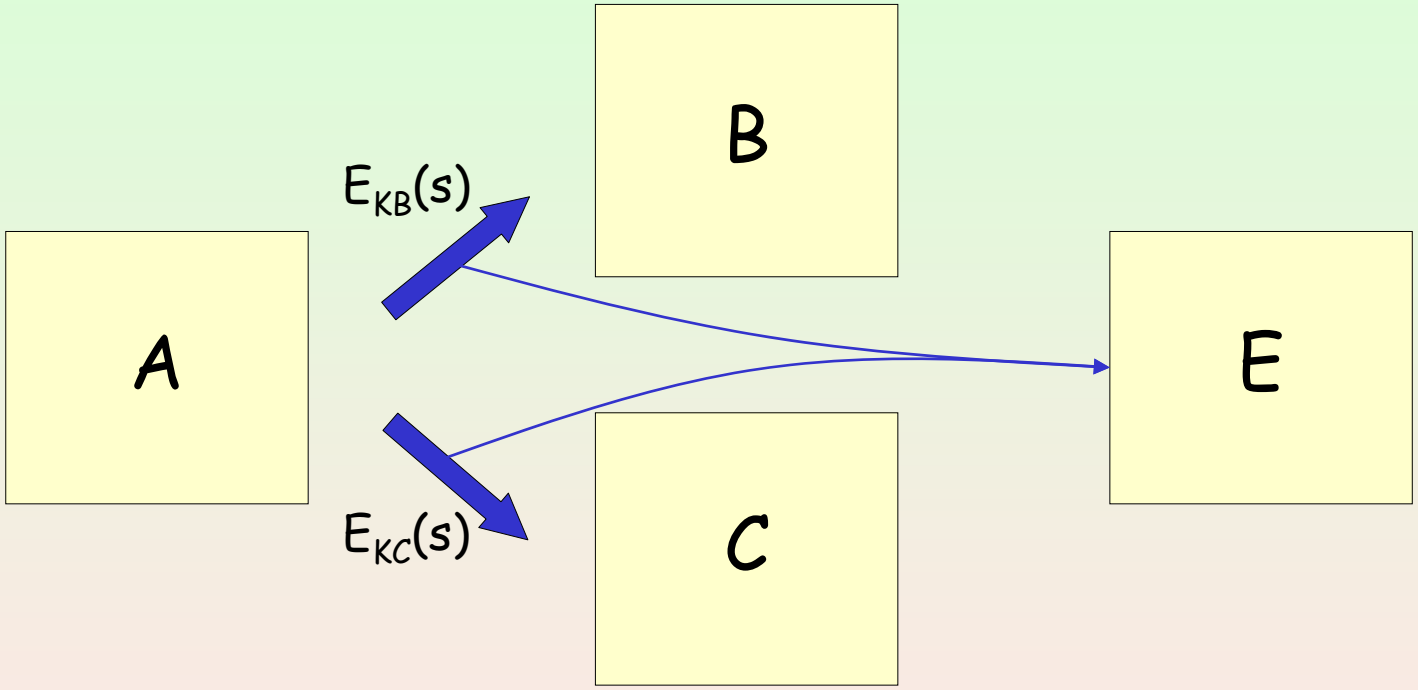
[Canetti, Cheung, Kaynar, Liskov, Lynch, Pereira, Segala, Vaandrager]



Nondeterminism: why There?

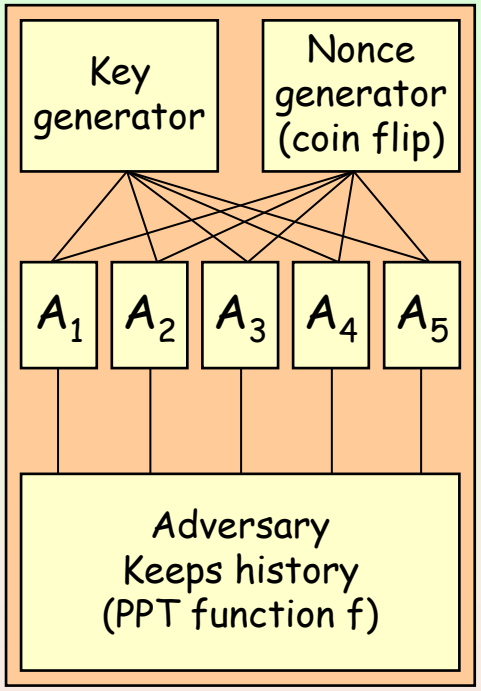
- If we have several components
  - Who moves first (nondeterminism)?
  - Can the order of operations reveal secrets?
- If we expect input
  - What input do we receive?
- If we have partial specification
  - How do we implement (nondeterminism)?
- Nondeterminism resolved by a "scheduler"
  - Not all resolutions are safe

Example of Nondeterminism



- Order of messages may reveal one bit of s to E

Example of Nondeterminism MAP1 Protocol [BR93]

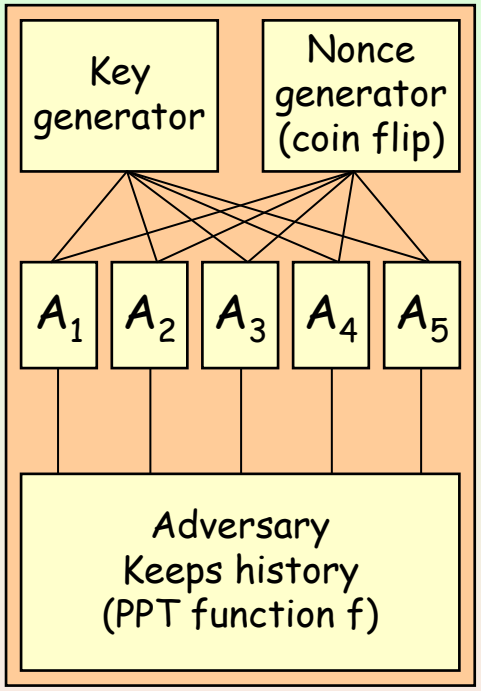


- Authentication protocol
  - Symmetric key signature schema
  - Computational Dolev-Yao
  - Adversary queries agents
- Potential problems
  - Let s be the shared key
  - Adversary queries k agents
  - Agent i replies if i^{th} bit of s is 1
  - The adversary knows the shared key

Approaches to Nondeterminism

- Reduce the power of the scheduler
  - Process Algebras
    - Scheduler sees only enabled action type
  - UC framework
    - ITMs have a token passing mechanism
    - No nondeterminism
  - Reactive simulatability
    - Again token passing mechanism
    - Nondeterminism based on local information only
  - Task PIOAs
    - Define equivalence classes of states and actions
    - Scheduler sees only equivalence classes, not elements
  - Careful specifications
    - Avoid dangerous nondeteminism in the specification
    - Is it always possible?

Example of Nondeterminism MAP1 Protocol [BR93]



- Authentication protocol
  - Symmetric key signature schema
  - Computational Dolev-Yao
  - Adversary queries agents
- Potential problems
  - Let s be the shared key
  - Adversary queries k agents
  - Agent i replies if i^{th} bit of s is 1
  - The adversary knows the shared key
- Solution
  - One query at a time
  - Wait for the answer (oracle model)

Concluding Remarks

- Formal methods are useful
  - Semi-formal proofs may be wrong
  - Semi-formal proofs require attention
- There are several approaches
  - Computational, symbolic, compositional
  - Suitable for cryptoprimitives or protocols
- We need proof techniques
  - Algebraic, symbolic, automata theoretic
- Nondeterminism arises and gives problems
  - Restrict resolution of nondeterminism
  - Avoid dangerous nondeterminism

What Else?

- Need more techniques
  - Can we have a uniform view
  - Can we relate better computational and symbolic approaches?
  - Any crucial differences between cryptoprimitives and protocols?
  - How about cross migration of techniques?
- Need more automation
  - … but we need to understand what we automate