Formal Verification of Cryptographic Protocols in Spi-Calculus

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Caution

- Literature on spi-calculus is confusing
  - Inconsistent terminology
  - Some "results" found too weak or even wrong
- This talk is my own combination of various results on spi-calculus

Outline

- What is spi-calculus?
  - Syntax and operational semantics
- Example protocol
- Attack against the example protocol
- Formalizing secrecy by non-interference
- Proving secrecy by hedged bisimulations
- Conclusions

What is spi-calculus? [Abadi-Gordon 99]

- spi-calulus = π-calculus + (shared-key) perfect encryption primitives

The only equation is:

$$\mathrm{dec}(\mathrm{enc}(\mathrm{Msg}, \mathrm{key}), \mathrm{key}) = \mathrm{Msg}$$

Cf. Textbook RSA is malleable:

$$\mathrm{enc}(\mathrm{Msg}_1, \mathrm{pubkey}) \times \mathrm{enc}(\mathrm{Msg}_2, \mathrm{pubkey}) = \mathrm{enc}(\mathrm{Msg}_1 \times \mathrm{Msg}_2, \mathrm{pubkey})$$

Syntax

$M, N ::=$	message
x	name
$\{M_1, \ldots, M_n\}_N$	ciphertext
$P, Q, R ::=$	process
0	inaction
$\overline{M}\langle N\rangle.P$	sending
$M(x).P$	receiving
$P \mid Q$	parallel composition
$(\nu x)P$	restriction
$!P$	replication
$\texttt{case}\ M\ \texttt{of}\ \{x_1, \ldots, x_n\}_N\ \texttt{in}\ P$	decryption
$[M = N]P$	matching

Operational Semantics (1/2): Structural Equivalence

$$\texttt{case}\ \{M_1,\ldots,M_n\}_N\ \texttt{of}\ \{x_1,\ldots,x_n\}_N\ \texttt{in}\ P$$
$$\equiv [M_1,\ldots,M_n/x_1,\ldots,x_n]P$$

$$[M = M]P \equiv P \qquad !P \equiv P \mid !P$$

$$P \mid (\nu x)Q \equiv (\nu x)(P \mid Q) \quad \text{if } x \notin \mathit{free}(P)$$

$$P \mid 0 \equiv P \qquad P \mid Q \equiv Q \mid P \qquad (P \mid Q) \mid R \equiv P \mid (Q \mid R)$$

$$\frac{P \equiv P'}{P \mid Q \equiv P' \mid Q} \qquad \frac{P \equiv P'}{(\nu x)P \equiv (\nu x)P'}$$

$$P \equiv P \qquad \frac{P \equiv Q}{Q \equiv P} \qquad \frac{P \equiv Q \quad Q \equiv R}{P \equiv R}$$

Operational Semantics (2/2): Reaction Relation

$$\overline{x}\langle M\rangle.P \mid x(y).Q \rightarrow P \mid [M/y]Q$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

$$\frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q} \qquad \frac{P \rightarrow P'}{(\nu x)P \rightarrow (\nu x)P'}$$

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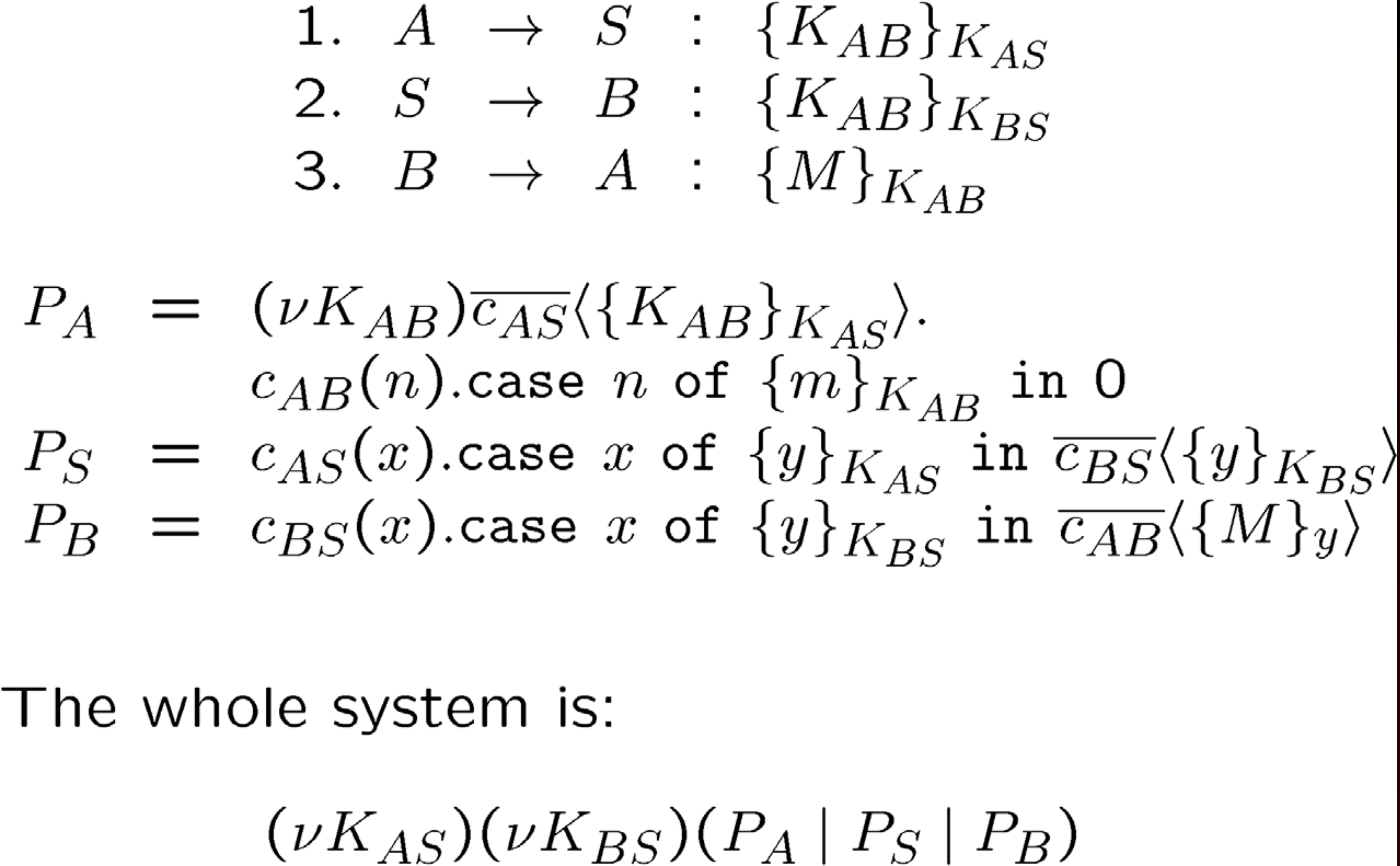
Example: A Naive Protocol (Wide Mouthed Frog Protocol)



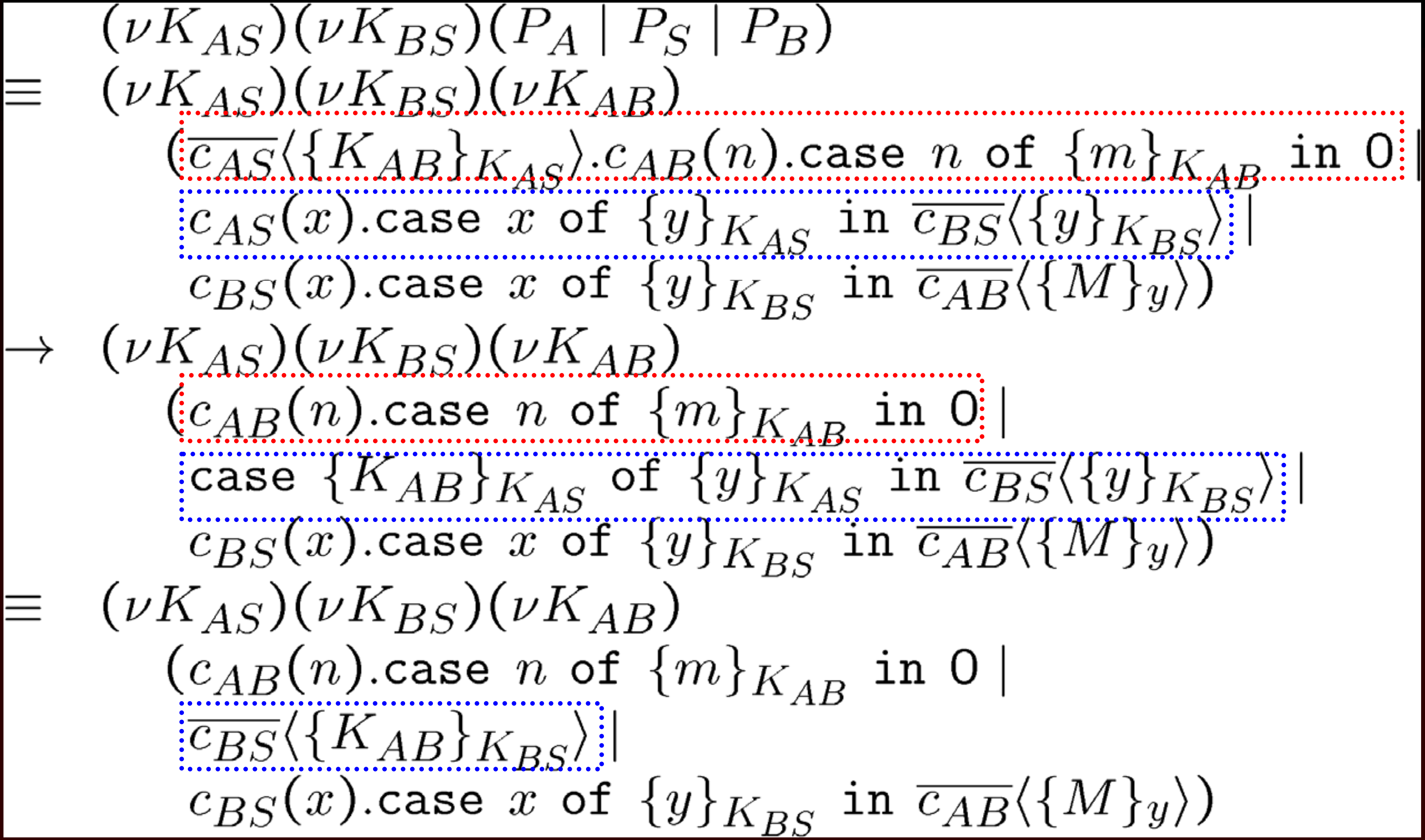
$$\begin{array}{llcll} 1. & A & \rightarrow & S & : \{K_{AB}\}_{K_{AS}} \\ 2. & S & \rightarrow & B & : \{K_{AB}\}_{K_{BS}} \\ 3. & B & \rightarrow & A & : \{M\}_{K_{AB}} \end{array}$$

$$\begin{array}{lcl} P_A & = & (\nu K_{AB})\overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle. \\ & & c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0 \\ P_S & = & c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{AS}}\ \mathtt{in}\ \overline{c_{BS}}\langle\{y\}_{K_{BS}} \\ P_B & = & c_{BS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{BS}}\ \mathtt{in}\ \overline{c_{AB}}\langle\{M\}_{y}\rangle \end{array}$$

The whole system is:

$$(\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B)$$

How does the protocol run? (1/2)



$$
\begin{array}{ll}
 & (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B) \\
\equiv & (\nu K_{AS})(\nu K_{BS})(\nu K_{AB}) \\
 & \quad (\boxed{\overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle.c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0} \mid \\
 & \quad \boxed{c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{AS}}\ \mathtt{in}\ \overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle} \mid \\
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How does the protocol run? (2/2)

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Parallel runs of the protocol (1/2)

$$\begin{array}{lllll}
1. & A & \rightarrow & S & : & \{K_{AB}\}_{K_{AS}} \\
2. & S & \rightarrow & B & : & \{K_{AB}\}_{K_{BS}} \\
3. & B & \rightarrow & A & : & \{M\}_{K_{AB}}
\end{array}$$

$$\begin{array}{llllll}
1'. & B & \rightarrow & S & : & \{K_{BE}\}_{K_{BS}} \\
2'. & S & \rightarrow & E & : & \{K_{BE}\}_{K_{ES}} \\
3'. & E & \rightarrow & B & : & \{M'\}_{K_{BE}}
\end{array}$$

Parallel runs of the protocol (2/2)

$$
\begin{aligned}
P_A &= (\nu K_{AB})\overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle. \\
&\quad c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0 \\
P_S &= c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{AS}}\ \mathtt{in}\ \overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle \\
&\mid\ c'_{BS}(x').\mathtt{case}\ x'\ \mathtt{of}\ \{y'\}_{K_{BS}}\ \mathtt{in}\ \overline{c_{ES}}\langle\{y'\}_{K_{ES}}\rangle \\
P_B &= c_{BS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{BS}}\ \mathtt{in}\ \overline{c_{AB}}\langle\{M\}_y\rangle \\
&\mid\ (\nu K_{BE})\overline{c'_{BS}}\langle\{K_{BE}\}_{K_{BS}}\rangle. \\
&\quad c_{BE}(n').\mathtt{case}\ n'\ \mathtt{of}\ \{m'\}_{K_{BE}}\ \mathtt{in}\ 0 \\
P_E &= c_{ES}(x').\mathtt{case}\ x'\ \mathtt{of}\ \{y'\}_{K_{ES}}\ \mathtt{in}\ \overline{c_{BE}}\langle\{M'\}_{y'}\rangle
\end{aligned}
$$

Exercise (?)

- Write down the reduction of $(\nu K_{AS})(\nu K_{BS})(\nu K_{ES})(P_A \mid P_S \mid P_B \mid P_E)$.

What if E is evil in fact?

- Assumption: attacker has full access to open channels (Dolev-Yao model)
- Result: not only M' but also M may leak!

$1'_a.\ B \rightarrow E(S)\ :\ \{K_{BE}\}_{K_{BS}}$

$2.\ E(S) \rightarrow B\ :\ \{K_{BE}\}_{K_{BS}}$

$1'_b.\ E(B) \rightarrow S\ :\ \{K_{BE}\}_{K_{BS}}$

$2'.\ S \rightarrow E\ :\ \{K_{BE}\}_{K_{ES}}$

$3.\ B \rightarrow E(A)\ :\ \{M\}_{K_{BE}}$

How does the attack work?

$$\begin{aligned}
P'_E \;=\; & c'_{BS}(z).\overline{c_{BS}}\langle z\rangle.\overline{c'_{BS}}\langle z\rangle. \\
& c_{ES}(x').\mathtt{case}\ x'\ \mathtt{of}\ \{y'\}_{K_{ES}}\ \mathtt{in} \\
& c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{y'}\ \mathtt{in}\ \mathsf{DoEvil}_m
\end{aligned}$$

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& P'_E \mid (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B) \\
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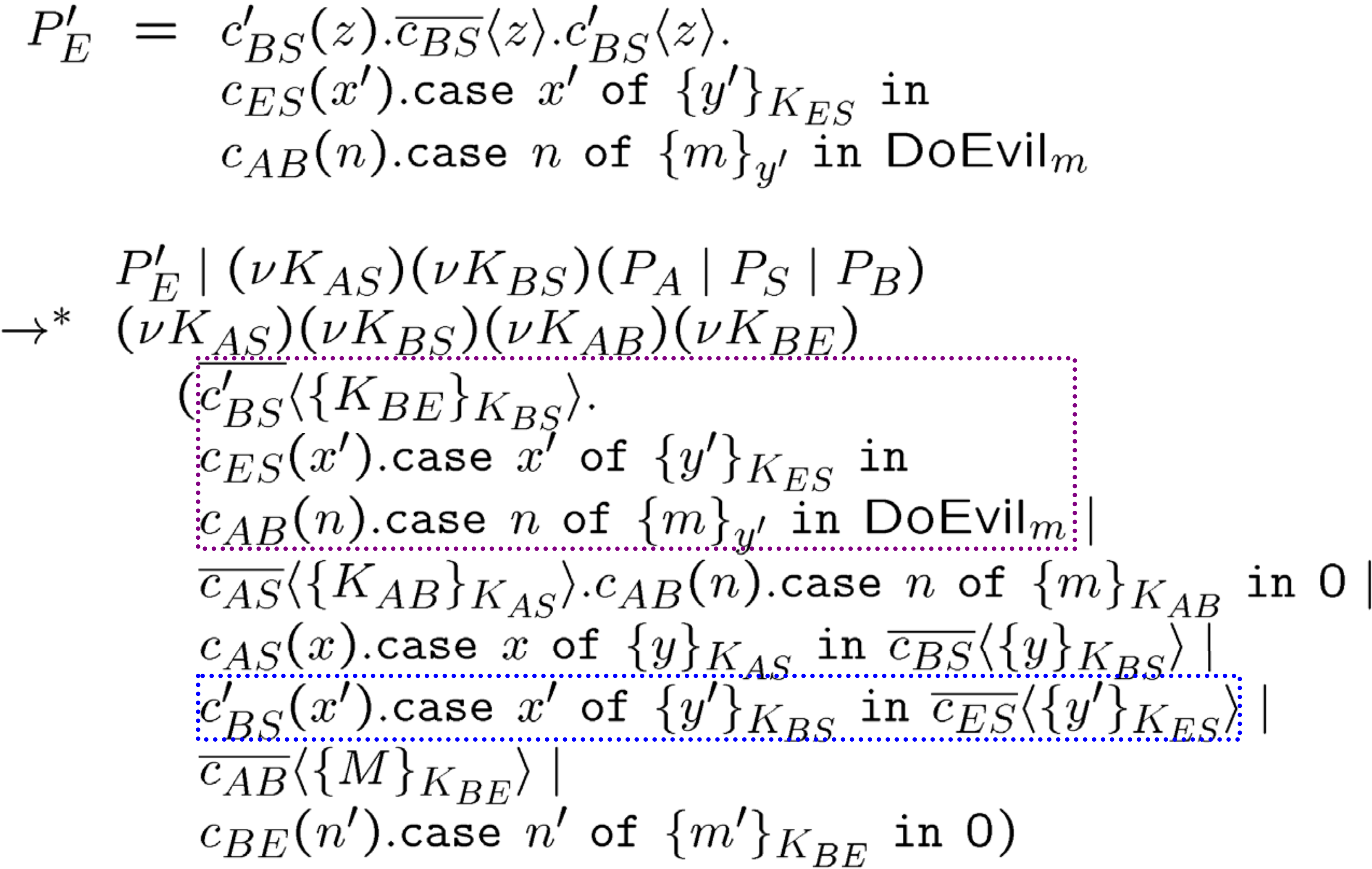

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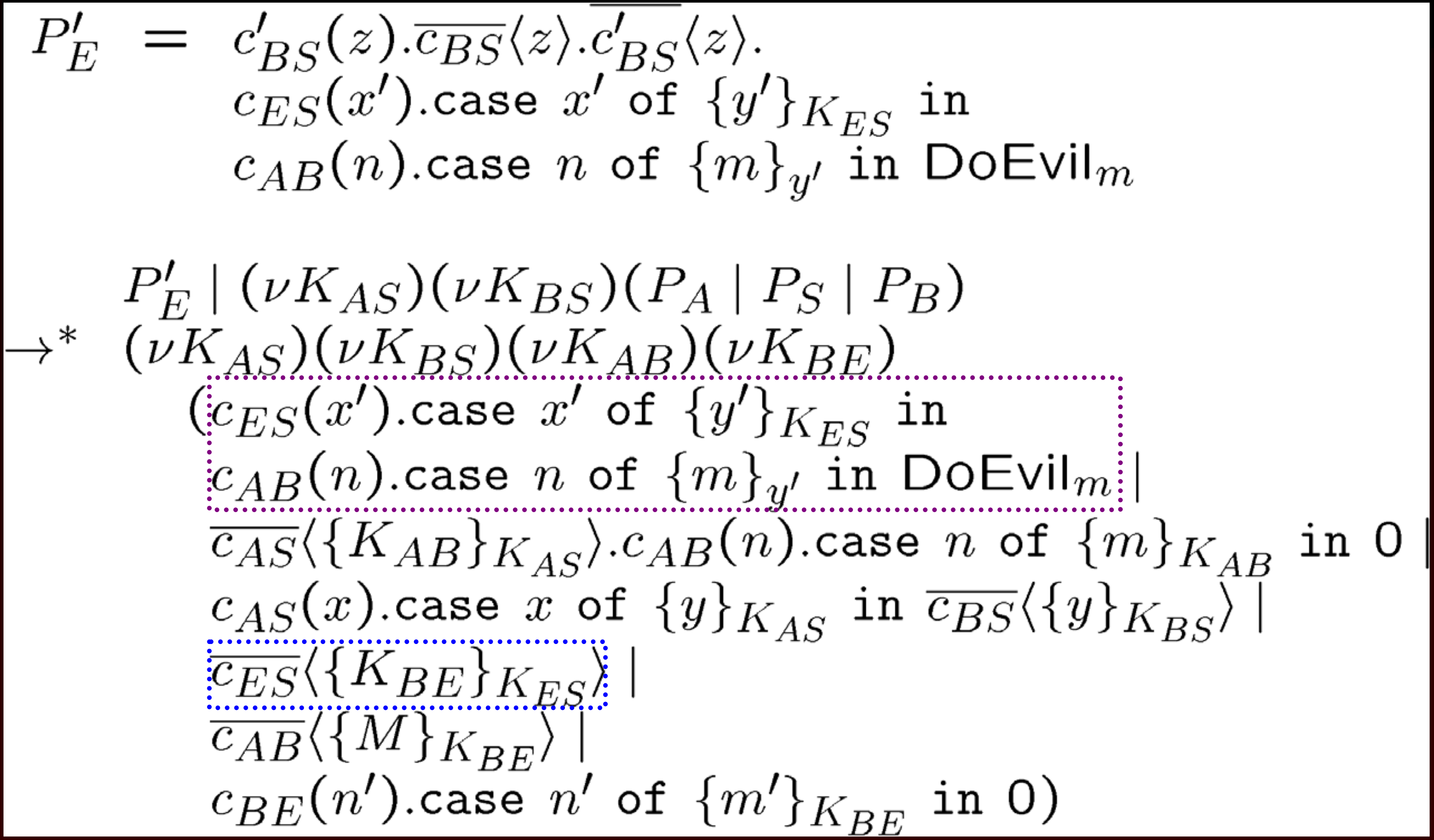

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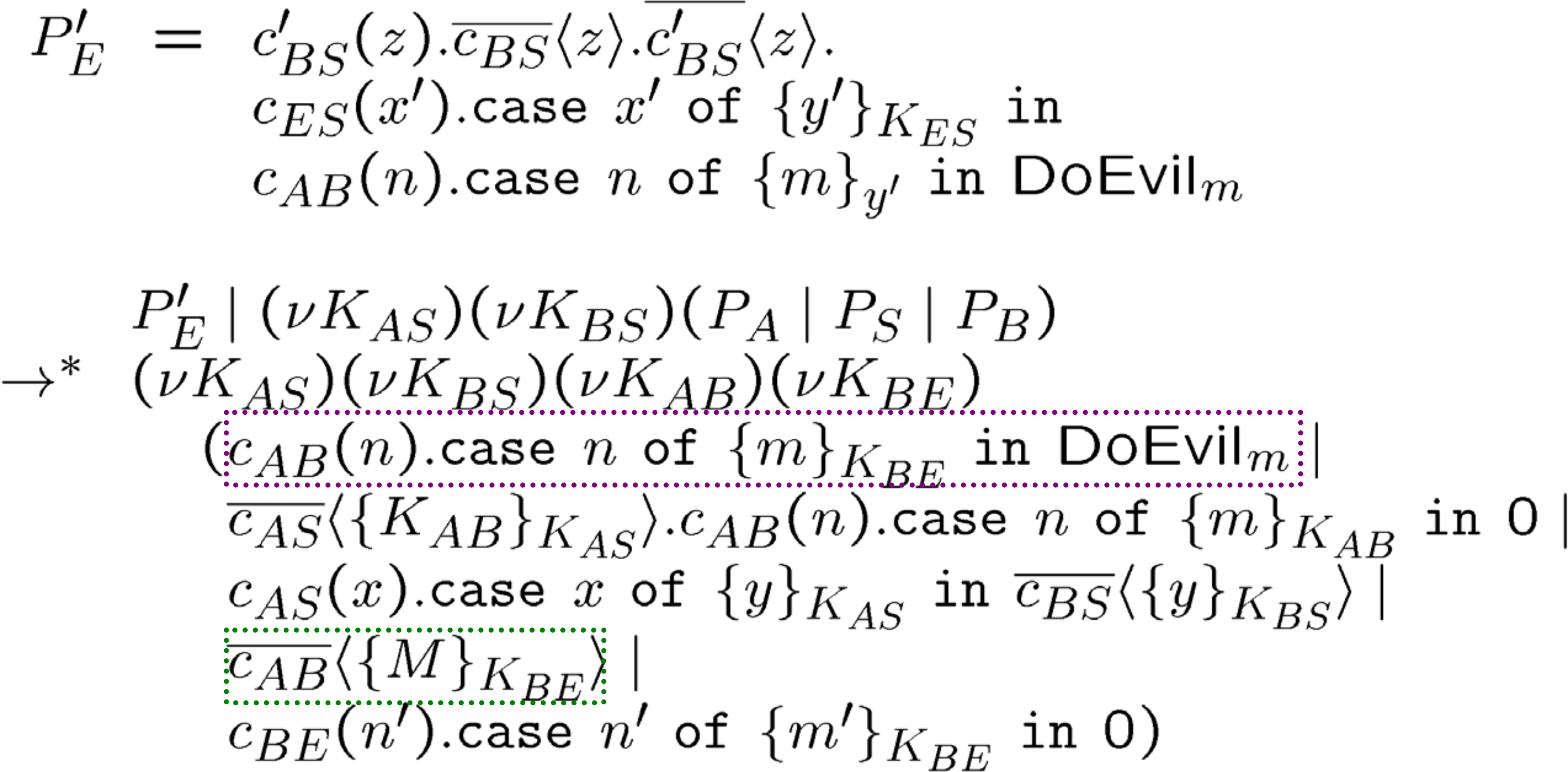

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& P'_E \mid (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B)\\
\rightarrow^{*}\;& (\nu K_{AS})(\nu K_{BS})(\nu K_{AB})(\nu K_{BE})\\
& \quad (\boxed{c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{BE}}\ \mathtt{in}\ \mathsf{DoEvil}_m} \mid\\
& \quad \overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle.c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0 \mid\\
& \quad c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{AS}}\ \mathtt{in}\ \overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle \mid\\
& \quad \boxed{\overline{c_{AB}}\langle\{M\}_{K_{BE}}\rangle} \mid\\
& \quad c_{BE}(n').\mathtt{case}\ n'\ \mathtt{of}\ \{m'\}_{K_{BE}}\ \mathtt{in}\ 0)
\end{aligned}
$$

How does the attack work?

$$\begin{aligned} P'_E \;=\;& c'_{BS}(z).\overline{c_{BS}}\langle z\rangle.\overline{c'_{BS}}\langle z\rangle. \\ & c_{ES}(x').\mathtt{case}\ x'\ \mathtt{of}\ \{y'\}_{K_{ES}}\ \mathtt{in} \\ & c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{y'}\ \mathtt{in}\ \mathsf{DoEvil}_m \end{aligned}$$

$$\begin{aligned} & P'_E \mid (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B) \\ \to^* \;& (\nu K_{AS})(\nu K_{BS})(\nu K_{AB})(\nu K_{BE}) \\ & \quad (\boxed{\mathsf{DoEvil}_M} \mid \\ & \quad \overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle.c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0 \mid \\ & \quad c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y\}_{K_{AS}}\ \mathtt{in}\ \overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle \mid \\ & \quad c_{BE}(n').\mathtt{case}\ n'\ \mathtt{of}\ \{m'\}_{K_{BE}}\ \mathtt{in}\ 0) \end{aligned}$$

Outline

- What is spi-calculus?
  - Syntax and operational semantics
- Example protocol
- Attack against the example protocol
- Formalizing secrecy by non-interference
- Proving secrecy by hedged bisimulations
- Conclusions

Formalizing secrecy by non-interference

- "Definition": Process P keeps message x totally secret if [M/x]P and [N/x]P are "equivalent" for any M and N

  Cf. partial secrecy: [M/x]P and [N/x]P are equivalent for any M and N satisfying some condition (e.g., M mod 2 = N mod 2)

- What equivalence should we take?
  ⇒ (Strong) barbed equivalence

Definitions (1/2)

- Process P immediately exhibits input barb c, written $P \downarrow c$, if

  $$P \equiv (\nu x_1)\ldots(\nu x_n)(c(y).Q \mid R)$$

  for some x_1, ..., x_n (distinct from c), y, Q and R. Similar for output.
- A (strong) barbed simulation S is a binary relation on processes such that $P \; S \; Q$ implies:
  - for each barb β, if $P \downarrow \beta$, then $Q \downarrow \beta$, and
  - if $P \rightarrow P'$, then $Q \rightarrow Q'$ and $P' \; S \; Q'$ for some Q
- S is a barbed bisimulation if both S and S^{-1} are barbed simulations

Definitions (2/2)

- Barbed <u>bisimilarity</u> is the largest barbed bisimulation
  - Equals the union of all barbed bisimulations, which is also a barbed bisimulation
- Processes P and Q are <u>barbed equivalent</u> if P | R and Q | R are barbed bisimilar for every R

Example

- $(\nu k)\overline{c}\langle\{x\}_k\rangle$ keeps x totally secret. I.e., $(\nu k)\overline{c}\langle\{M\}_k\rangle$ and $(\nu k)\overline{c}\langle\{N\}_k\rangle$ are barbed equivalent for any M and N.

Proof sketch: given M and N, take

$$\begin{aligned} S = \{ (P, Q) \mid\ & P \equiv (\nu k)\ [\{M\}_k/y]R, \\ & Q \equiv (\nu k)\ [\{N\}_k/y]R, \\ & k \notin \mathrm{free}(R) \ \} \end{aligned}$$

and prove it to be a barbed bisimulation by case analysis (and induction) on the reduction rules

Example

- $P = (\nu k)(\overline{c}\langle\{x\}_k\rangle \mid$
  $c(y).\mathtt{case}\ y\ of\ \{z\}_k\ in\ \overline{c}\langle k\rangle)$
  does not keep x totally secret. Indeed, [M/x]P and [N/x]P are not barbed equivalent for any M ≠ N.

Proof: given M and N, take

$$R \;=\; c(y).\overline{c}\langle y\rangle.c(k).\mathtt{case}\ y\ \mathtt{of}\ \{m\}_k\ \mathtt{in}\ [m = M]\overline{world}\langle hello\rangle$$

Cf. $P = (\nu k)(\overline{k}\langle x\rangle \mid k(y).\overline{c}\langle k\rangle)$ does keep x secret

Side Step: The Vice of May Testing Equivalence

- Many papers (including Abadi and Gordon's original work!) use may testing equivalence for defining secrecy by non-interference, but it is too weak

Definitions

- Process P may eventually exhibit barb β, written $P \Downarrow \beta$, if $P \rightarrow \ldots \rightarrow P' \downarrow \beta$ for some P'
- Processes P and Q are may testing equivalent if

$$(P \mid R) \Downarrow \beta \quad \Leftrightarrow \quad (Q \mid R) \Downarrow \beta$$

for every R and β

So what's wrong?

- Surprisingly,

$$P \quad = \quad (\nu d)(\bar{d}\langle\rangle \mid d().\bar{c}\langle\rangle)$$

and

$$Q \quad = \quad (\nu d)(\bar{d}\langle\rangle \mid d().\bar{c}\langle\rangle \mid d().0)$$

are may testing equivalent.

- As a result, processes like

if x > 0 then P else Q

are regarded as keeping x totally secret (under may testing equivalence)

- But the leak is possible!

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Hedged Bisimulation: Motivation

Direct proof of barbed equivalence is difficult because of "arbitrary R"

⇒ Devise a proof technique without "arbitrary R"

♦ What can R do?
  - Gain "knowledge" by receiving from a known channel
  - Send to a known channel a message synthesized from the knowledge

Definitions (1/4)

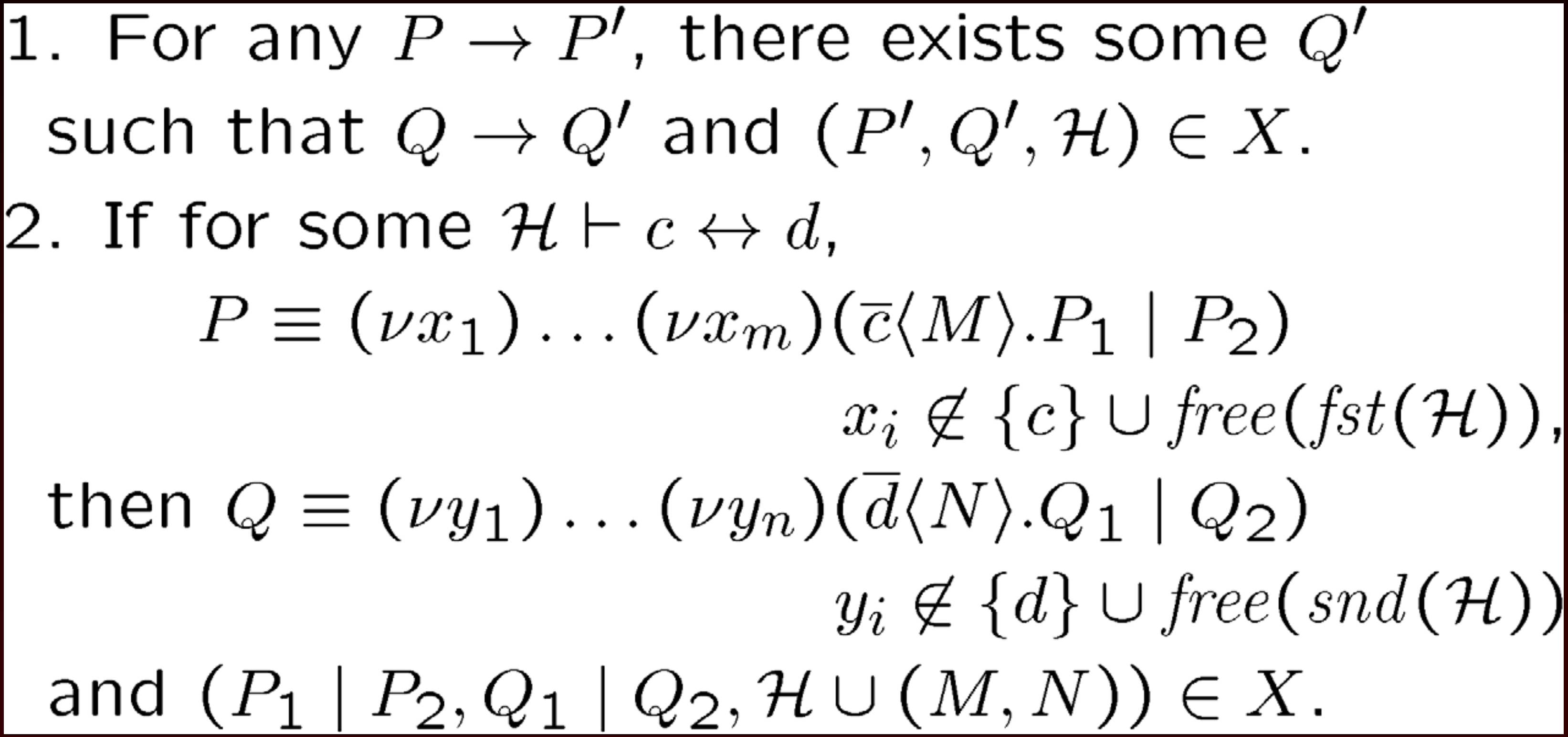
- A hedge H is a binary relation on messages
- H ⊢ M ↔ N (messages M and N can be synthesized from hedge H) is defined by induction:

$$\frac{(M,N)\in\mathcal{H}}{\mathcal{H}\vdash M\leftrightarrow N} \qquad \frac{\mathcal{H}\vdash M_1\leftrightarrow N_1 \quad \mathcal{H}\vdash M_2\leftrightarrow N_2}{\mathcal{H}\vdash \{M_1\}_{M_2}\leftrightarrow \{N_1\}_{N_2}}$$

$$\frac{\mathcal{H}\vdash \{M_1\}_{M_2}\leftrightarrow \{N_1\}_{N_2} \quad \mathcal{H}\vdash M_2\leftrightarrow N_2}{\mathcal{H}\vdash M_1\leftrightarrow N_1} \qquad \frac{x\notin free(\mathcal{H})}{\mathcal{H}\vdash x\leftrightarrow x}$$

Definitions (2/4)

- A hedged simulation is a set X of triples (P, Q, H) that satisfies:



1. For any $P \rightarrow P'$, there exists some Q' such that $Q \rightarrow Q'$ and $(P', Q', \mathcal{H}) \in X$.
2. If for some $\mathcal{H} \vdash c \leftrightarrow d$,

$$P \equiv (\nu x_1) \ldots (\nu x_m)(\overline{c}\langle M \rangle.P_1 \mid P_2)$$

$$x_i \notin \{c\} \cup \mathit{free}(\mathit{fst}(\mathcal{H})),$$

then $Q \equiv (\nu y_1) \ldots (\nu y_n)(\overline{d}\langle N \rangle.Q_1 \mid Q_2)$

$$y_i \notin \{d\} \cup \mathit{free}(\mathit{snd}(\mathcal{H}))$$

and $(P_1 \mid P_2, Q_1 \mid Q_2, \mathcal{H} \cup (M, N)) \in X$.

Definitions (3/4)

3. If for some $\mathcal{H} \vdash c \leftrightarrow d$,

$$P \equiv (\nu x_1) \ldots (\nu x_m)(c(z).P_1 \mid P_2)$$

$$x_i \notin \{c\} \cup \mathit{free}(\mathit{fst}(\mathcal{H})),$$

then $Q \equiv (\nu y_1) \ldots (\nu y_n)(d(z).Q_1 \mid Q_2)$

$$y_i \notin \{d\} \cup \mathit{free}(\mathit{snd}(\mathcal{H}))$$

and for any $\mathcal{H} \vdash M \leftrightarrow N$,

$$([M/z]P_1 \mid P_2, [N/z]Q_1 \mid Q_2, \mathcal{H}) \in X.$$

4. If $\mathcal{H} \vdash M_1 \leftrightarrow N_1$ and $\mathcal{H} \vdash M_2 \leftrightarrow N_2$, then $M_1 = M_2$ implies $N_1 = N_2$.

5. If $\mathcal{H} \vdash \{M_1\}_{M_2} \leftrightarrow N$ and $\mathcal{H} \vdash M_2 \leftrightarrow N_2$, then $N = \{N_1\}_{N_2}$ for some N_1.

Definitions (4/4)

- A hedged simulation X is a hedged bisimulation if X^{-1} is also a hedged simulation, where X^{-1} is defined as:

$$\{(Q, P, H^{-1}) \mid (P, Q, H) \in X\}$$

- Hedged bisimilarity is the largest hedged bisimulation (i.e., the union of all hedged bisimulations, which is also a hedged bisimulation)
- Notation: $P \sim_H Q \Leftrightarrow (P, Q, H)$ is in the hedged bisimilarity

Caution: α-Conversion of Hedged Bisimulation

- Every $(P, Q, H) \in X$ is regarded as α-equivalent to
  $$(\sigma P, Q, \{ (\sigma M, N) \mid (M, N) \in H \})$$
  for every $\mathrm{dom}(\sigma) \supseteq \mathrm{free}(P) \cup \mathrm{free}(\mathrm{fst}(H))$
- Every $(P, Q, H) \in X$ is regarded as α-equivalent to
  $$(P, \sigma Q, \{ (M, \sigma N) \mid (M, N) \in H \})$$
  for every $\mathrm{dom}(\sigma) \supseteq \mathrm{free}(Q) \cup \mathrm{free}(\mathrm{snd}(H))$
- Everything in the rest is considered "up to" this α-equivalence

Example 1

- For any M and N,

$$(\nu k)\overline{c}\langle\{M\}_k\rangle.0 \sim_{\{(c,c)\}} (\nu k)\overline{c}\langle\{N\}_k\rangle.0$$

Proof: take

$$\begin{array}{rcl} X & = & \{((\nu k)\overline{c}\langle\{M\}_k\rangle.0, \\ & & \quad (\nu k)\overline{c}\langle\{N\}_k\rangle.0, \\ & & \quad \{(c,c)\})\} \\ & \cup & \{(0, \\ & & \quad 0, \\ & & \quad \{(c,c),(\{M\}_k,\{N\}_k)\})\} \end{array}$$

and check conditions 1-5.

Example 2

- ◆ $(\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.(\nu m)\overline{c}\langle m\rangle \sim_{\{(c,c)\}} (\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.\overline{c}\langle n\rangle$

Proof: take

$$\begin{aligned}
X \quad = \quad & \{((\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.(\nu m)\overline{c}\langle m\rangle, \\
& \quad (\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.\overline{c}\langle n\rangle, \\
& \quad \{(c,c)\})\} \\
\cup \quad & \{((\nu m)\overline{c}\langle m\rangle, \\
& \quad \overline{c}\langle n\rangle, \\
& \quad \{(c,c),(\{n\}_k,\{n\}_k)\})\} \\
\cup \quad & \{(0, \\
& \quad 0, \\
& \quad \{(c,c),(\{n\}_k,\{n\}_k),(m,n)\})\}.
\end{aligned}$$

Example 3

- $(\nu k)(\nu n)(\nu l)\overline{c}\langle\{\{n\}_k\}_l\rangle.(\nu m)\overline{c}\langle m\rangle \sim_{\{(c,c)\}} (\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.(\nu m)\overline{c}\langle m\rangle$

Proof: take

$$\begin{array}{rcl}
X & = & \{((\nu k)(\nu n)(\nu l)\overline{c}\langle\{\{n\}_k\}_l\rangle.(\nu m)\overline{c}\langle m\rangle, \\
& & \quad (\nu k)(\nu n)\overline{c}\langle\{n\}_k\rangle.(\nu m)\overline{c}\langle m\rangle, \\
& & \quad \{(c,c)\})\} \\
& \cup & \{((\nu m)\overline{c}\langle m\rangle, \\
& & \quad (\nu m)\overline{c}\langle m\rangle, \\
& & \quad \{(c,c),(\{\{n\}_k\}_l,\{n\}_k)\})\} \\
& \cup & \{(0, \\
& & \quad 0, \\
& & \quad \{(c,c),(\{\{n\}_k\}_l,\{n\}_k),(m,m)\})\}.
\end{array}$$

Theorem

Hedged bisimilarity is sound w.r.t. barbed equivalence. I.e., if $P \sim_H Q$ for

$$H = \{ (x, x) \mid x \in \mathrm{free}(P) \cup \mathrm{free}(Q) \},$$

then P and Q are barbed equivalent.

Proof sketch: take

$$S = \{ (P', Q') \mid P \sim_H Q,$$
$$P' \equiv (\nu x_1)\ldots(\nu x_l)\ (P \mid [M_1,\ldots,M_n/z_1,\ldots,z_n]R),$$
$$Q' \equiv (\nu y_1)\ldots(\nu y_m)\ (Q \mid [N_1,\ldots,N_n/z_1,\ldots,z_n]R),$$
$$H \vdash M_1 \leftrightarrow N_1,\ \ldots,\ H \vdash M_n \leftrightarrow N_n,$$
$$\mathrm{free}(R) \text{ distinct from } \mathrm{free}(P), \mathrm{free}(Q), \text{ and } \mathrm{free}(H)) \}$$

and prove it to be a barbed bisimulation by case analysis (and induction) on the reduction rules.

Real Example: Fixed Version of Previous Protocol

$$\begin{array}{lllll}
1. & A & \rightarrow & S & : \{K_{AB}, B\}_{K_{AS}} \\
2. & S & \rightarrow & B & : \{K_{AB}, A\}_{K_{BS}} \\
3. & B & \rightarrow & A & : \{M\}_{K_{AB}}
\end{array}$$

$$\begin{array}{lllll}
1'. & B & \rightarrow & S & : \{K_{BE}, E\}_{K_{BS}} \\
2'. & S & \rightarrow & E & : \{K_{BE}, B\}_{K_{ES}} \\
3'. & E & \rightarrow & B & : \{M'\}_{K_{BE}}
\end{array}$$

As Spi-Calculus Processes...

$$
\begin{array}{rcl}
P_A & = & (\nu K_{AB})\overline{c_{AS}}\langle\{K_{AB}, B\}_{K_{AS}}\rangle. \\
 & & c_{AB}(n).\mathtt{case}\ n\ \mathtt{of}\ \{m\}_{K_{AB}}\ \mathtt{in}\ 0 \\
P_S & = & c_{AS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y, b\}_{K_{AS}}\ \mathtt{in} \\
 & & [b = B]\overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle \\
 & | & c'_{BS}(x').\mathtt{case}\ x'\ \mathtt{of}\ \{y', e\}_{K_{BS}}\ \mathtt{in} \\
 & & [e = E]\overline{c_{ES}}\langle\{y'\}_{K_{ES}}\rangle \\
P_B & = & c_{BS}(x).\mathtt{case}\ x\ \mathtt{of}\ \{y, a\}_{K_{BS}}\ \mathtt{in} \\
 & & [a = A]\overline{c_{AB}}\langle\{z\}_y\rangle \\
 & | & (\nu K_{BE})c'_{BS}\langle\{K_{BE}, E\}_{K_{BS}}\rangle. \\
 & & c_{BE}(n').\mathtt{case}\ n'\ \mathtt{of}\ \{m'\}_{K_{BE}}\ \mathtt{in}\ 0
\end{array}
$$

Exercise (?)

- Write down the reduction(s) of
  $$P'_E \mid (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B)$$
  for the same attacker P'_E as before, for the fixed version of P_A, P_S, and P_B. Pinpoint where the attack fails.

Claim

- $(\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid P_B)$
  keeps z totally secret. I.e.,
  $P = (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid [M/z]P_B)$
  and
  $Q = (\nu K_{AS})(\nu K_{BS})(P_A \mid P_S \mid [N/z]P_B)$
  are barbed equivalent for any M and N.

Proof Sketch

- Let $H = \{ (x, x) \mid x \in \mathrm{free}(P) \cup \mathrm{free}(Q) \}$
- We construct some hedged bisimulation $X \ni (P, Q, H)$
  - The X is far from minimal, but this is fine as far as X is a hedged bisimulation
    - It is a nightmare to write down minimal X for real...

$$P_A = (\nu K_{AB})\,\overbrace{\overline{c_{AS}}\langle\{K_{AB},B\}_{K_{AS}}\rangle.\,\overbrace{c_{AB}(n).\,\overbrace{\texttt{case}\ n\ \texttt{of}\ \{m\}_{K_{AB}}\ \texttt{in}\ 0}^{P_{A_2}}}^{P_{A_1}}}^{P_{A_0}}$$

$$\begin{aligned}
P_S &= \overbrace{c_{AS}(x).\,\overbrace{\texttt{case}\ x\ \texttt{of}\ \{y,b\}_{K_{AS}}\ \texttt{in}\ [b=B]\,\overbrace{\overline{c_{BS}}\langle\{y\}_{K_{BS}}\rangle}^{P_{S_3}}}^{P_{S_1}}}^{P_{S_0}} \\
&\mid\ \underbrace{c'_{BS}(x').\,\underbrace{\texttt{case}\ x'\ \texttt{of}\ \{y',e\}_{K_{BS}}\ \texttt{in}\ \underbrace{[e=E]\,\underbrace{\overline{c_{ES}}\langle\{y'\}_{K_{ES}}\rangle}_{P'_{S_3}}}_{P'_{S_2}}}_{P'_{S_1}}}_{P'_{S_0}}
\end{aligned}$$

$$
\begin{aligned}
P_B \;=\;& \overbrace{c_{BS}(x).\,\overbrace{\mathtt{case}\ x\ \mathtt{of}\ \{y,a\}_{K_{BS}}\ \mathtt{in}\ \overbrace{[a=A]\,\overbrace{\overline{c_{AB}}\langle\{z\}_y\rangle}^{P_{B_3}}}^{P_{B_2}}}^{P_{B_1}}}^{P_{B_0}} \\
\mid\;& \underbrace{(\nu K_{BE})\,\underbrace{\overline{c'_{BS}}\langle\{K_{BE},E\}_{K_{BS}}\rangle.c_{BE}(n').\,\underbrace{\mathtt{case}\ n'\ \mathtt{of}\ \{m'\}_{K_{BE}}\ \mathtt{in}\ 0}_{P'_{B_2}}}_{P'_{B_1}}}_{P'_{B_0}}
\end{aligned}
$$

$X = \{ (P', Q', H') \mid$

$P' \equiv (\nu c_1)\ldots(\nu c_u)$

$([M_1/n]P_{A_i} \mid [M_2/x]P_{S_j} \mid [M_3,A/x',e]P'_{S_k} \mid$

$[M_4,E,M/x,a,z]P_{B_l} \mid [M_5/n']P'_{B_m}),$

$Q' \equiv (\nu d_1)\ldots(\nu d_v)$

$([N_1/n]P_{A_i} \mid [N_2/x]P_{S_j} \mid [N_3,A/x',e]P'_{S_k} \mid$

$[N_4,E,N/x,a,z]P_{B_l} \mid [N_5/n']P'_{B_m}),$

$H' \subseteq H \cup \{ (\{K_{AB},B\}_{K_{AS}}, \{K_{AB},B\}_{K_{AS}}),$

$(\{K_{AB}, A\}_{K_{BS}}, \{K_{AB}, A\}_{K_{BS}}),$

$(\{M\}_{K_{AB}}, \{N\}_{K_{AB}}),$

$(\{K_{BE}, E\}_{K_{BS}}, \{K_{BE}, E\}_{K_{BS}}),$

$(\{K_{BE}, B\}_{K_{ES}}, \{K_{BE}, B\}_{K_{ES}}) \},$

$H' \vdash M_w \leftrightarrow N_w$ for $w = 1, 2, 3, 4, 5,$

$c_1, \ldots, c_u \notin \mathrm{free}(\mathrm{fst}(H')),$

$d_1, \ldots, d_v \notin \mathrm{free}(\mathrm{snd}(H')) \}$

Exercise (?)

- Try to prove the total secrecy of z in the original version of this protocol by means of hedged bisimulation. Explain how the "proof" fails.

Side Step II: Completeness of Hedged Bisimulation

Conjecture:

Hedged bisimilarity is complete with respect to barbed equivalence.

I.e., if P and Q are barbed equivalent, then $P \sim_H Q$ for

$$H = \{ (x, x) \mid x \in \mathrm{free}(P) \cup \mathrm{free}(Q) \}$$

- Proved for "structurally image finite" processes, but not for the general case (to my knowledge)

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Other Topics in Spi-Calculus

- Other bisimulations [Abadi-Gordon 98] [Boreale-DeNicola-Pugliese 99] [Elkjær-Höhle-Hüttel-Overgård 99]
  - More complex and "less complete"
- Secrecy by typing [Abadi 97] [Abadi-Blanchet 01]
- Authenticity by typing [Gordon-Jeffery 01] [Gordon-Jeffery 02] [Blanchet 02]

Cf. http://www.soe.ucsc.edu/~abadi/
http://www.di.ens.fr/~blanchet/
http://netlib.bell-labs.com/who/ajeffrey/ etc.