Using Task-Structured PIOAs to Analyze Cryptographic Protocols

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Motivation

Make:

- systematic proofs
- in a composable security setting
- considering probabilistic and nondeterministic behaviors
- including nondeterministic protocol specification

Nondeterministic behavior

Why? Experience from concurrency theory says:

- just specify what is needed for the protocol to work
- simplicity: avoids "clutter" in the specification
- generality: keeps freedom for the implementer

Example: Oblivious Transfer

OT functionality without internal nondeterminism:

Version 1:

- on input (x_0, x_1) from T, store (x_0, x_1)
- on input i from R: if input (x_0, x_1) was received, send x_i to R, else do nothing

Example: Oblivious Transfer

OT functionality without internal nondeterminism:
Version 2:

- on input (x_0, x_1) from T: if input i was received, send x_i to R, else store (x_0, x_1)
- on input i from R: if input (x_0, x_1) was received, send x_i to R, else store i

OT functionality with internal nondeterminism:

- on input (x_0, x_1) from T, store (x_0, x_1)
- on input i from R, store i
- if (x_0, x_1) and i have been received, send x_i to R

Example: Needham-Schroeder-Lowe

Receiver role:

Version 1:

1. Receive $\{|N_a, A|\}_{K_B}$
2. Select N_b
3. Send $\{|N_a, N_b, B|\}_{K_A}$

Version 2:

1. Select N_b
2. Receive $\{|N_a, A|\}_{K_B}$
3. Send $\{|N_a, N_b, B|\}_{K_A}$

- from a security point of view: who cares?
- according to the hardware, one solution might be better than the other

Motivation

Make:

- systematic proofs
- in a composable setting
- exhibiting probabilistic and nondeterministic behaviors
- including in protocol specification

We want to prove security *for every way to resolve the nondeterminism*

This work...

In this work, we propose:

- a new model for the analysis of crypto protocols
  - protocols can have internal nondeterminism
  - enables simulation based security for nondeterministic systems
- an analysis of an Oblivious Transfer protocol [EGL85,GMW87] in our model

Starting Point

PIOAs [Seg95, LSV03] are kinds of interacting, abstract, automata:

- state variables
- actions (input, output, internal)
- transitions: $(\mathit{state} \times \mathit{action}) \rightarrow \mathrm{Disc}(\mathit{states}) \cup \bot$

Internal nondeterminism for output and internal actions

- not algorithmically resolved
- kept unresolved in the analyzed systems

Resolving nondeteminism

- PIOAs use schedulers with full knowledge of current state — way too powerful!
- We introduce *tasks*, i.e.,
  - equivalence classes on actions, abstracting from state variables (ex: send message 1, select key, . . .)
  - given a task, at most one possible (probabilistic) action
- We introduce *task schedulers*: just sequences of tasks
- Execution: read first task, find and execute the enabled action (if there is one), go to next task, . . .

Task-PIOAs

Task-PIOAs provide:

- realistic model of network behavior:
  - adversary (just one task-PIOA) still controls the network
  - adversary cannot decide when parties send messages
- freedom for implementers:
  - implementers can resolve internal nondeterminism as they want
  - not a concern in model or security proofs

Example: OT Functionality

Input actions:

$in(x)_{Trans}$, $x \in (\{0,1\} \to \{0,1\})$

$in(i)_{Rec}$, $i \in \{0,1\}$

Output actions:

$out(x)_{Rec}$, $x \in \{0,1\}$

State:

$inval(Trans) \in (\{0,1\} \to \{0,1\}) \cup \{\perp\}$, initially $\perp$

$inval(Rec) \in \{0,1,\perp\}$, initially $\perp$

Transitions:

$in(x)_{Trans}$: if $inval(Trans) = \perp$ then $inval(Trans) := x$

$in(i)_{Rec}$: if $inval(Rec) = \perp$ then $inval(Rec) := i$

$out(x)_{Rec}$: Preconditions: $inval(Trans), inval(Rec) \neq \perp$

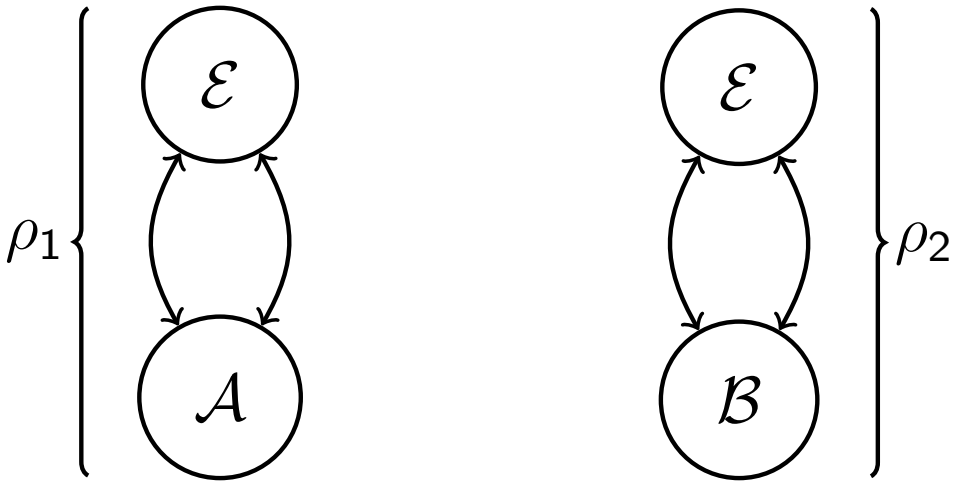
$x = inval(Trans)(inval(Rec))$

Tasks: $\{out(*)_{Rec}\}$

Indistinguishability

Implementation relation for task-PIOAs:

- $\mathcal{A} \leq \mathcal{B}$ means:
  $\forall$ environment $\mathcal{E}$ for $\mathcal{A}$ and $\mathcal{B}$,
  and $\forall$ task scheduler ρ_1 for $\mathcal{A}||\mathcal{E}$,
  $\exists$ task scheduler ρ_2 for $\mathcal{B}||\mathcal{E}$
  s.t. $\mathcal{E}$ cannot distinguish $\mathcal{A}$ from $\mathcal{B}$



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- Indistinguishability for nondeterminisitic systems
- $\leq$ transitive: $A \leq B,\ B \leq C \Rightarrow A \leq C$
- $\leq$ composable: $A \leq B \Rightarrow A||C \leq B||C$

Proving Security

Two variants of $\leq$:

- $\leq_0$, for perfect indistinguishability
- $\leq_{neg,pt}$ for computational indistinguishability

$\leq_0$ proved using a sound simulation relation

- $\approx$ matching (distributions on) states
- very systematic

$\leq_{neg,pt}$ proved using computational assumptions

- Express computational assumptions as $C_1 \leq_{neg,pt} C_2$
- Composition: $C_1 \leq_{neg,pt} C_2 \Rightarrow C_1||Ifc \leq_{neg,pt} C_2||Ifc$

Proving Security

Proofs are modular:

- $A \leq B$ proved as $A \leq A_1 \leq \cdots \leq A_n \leq B$
  - $\approx$ sequences of games, but for automata
- Composition properties allow reusing proofs for small systems in bigger ones

Reasoning at multiple abstraction levels:

- systematic proof techniques in terms of task-PIOAs
- can be used at computational level *and* at D-Y level

Establishing $C_1 \leq_{neg,pt} C_2$

Example: Hard-core predicates for trapdoor permutations

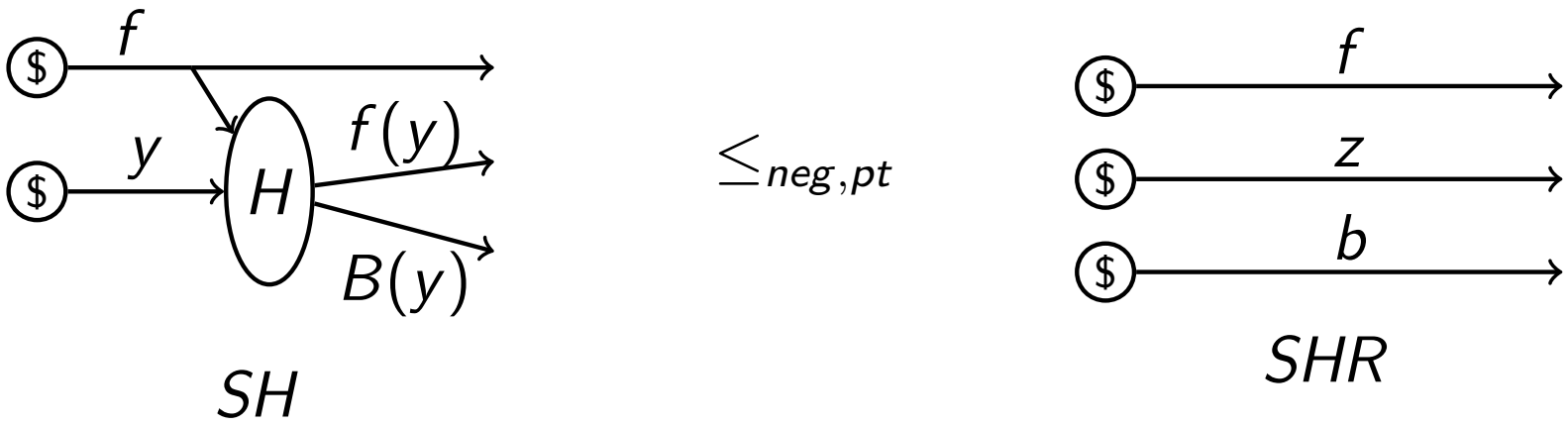
Crypto: for every PPT G, there is a negligible ϵ:

$$\left| \begin{array}{l} \Pr[f \leftarrow Tdp; \\ \quad y \leftarrow D; \\ \quad b \leftarrow B(y) : \\ \quad G(f, f(y), b) = 1] \end{array} - \begin{array}{l} \Pr[f \leftarrow Tdp; \\ \quad z \leftarrow D; \\ \quad b \leftarrow \{0,1\} : \\ \quad G(f, z, b) = 1] \end{array} \right| \leq \epsilon$$

Defining H-C Predicates in terms of PIOAs

We transpose this classical crypto assumption to task-PIOAs.

$SH \leq_{neg,pt} SHR$:



Theorem: Both formulations are equivalent!

Using Computational Assumptions

What's happening if we use 2 hard-core bits?

In some protocol, we:

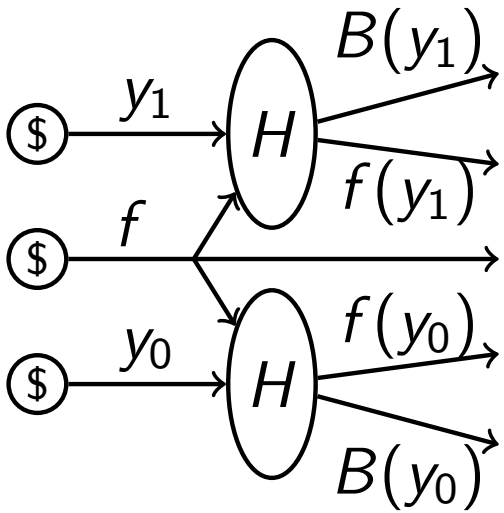
- select one trapdoor permutation f
- select two elements of the domain of f, say, (y_0, y_1)
- transmit $f(y_0), f(y_1), B(y_0), B(y_1)$

Do we keep the same indistinguishability guarantee?

- That is, $B(y_0)$ and $B(y_1)$ cannot be distinguished from random bits

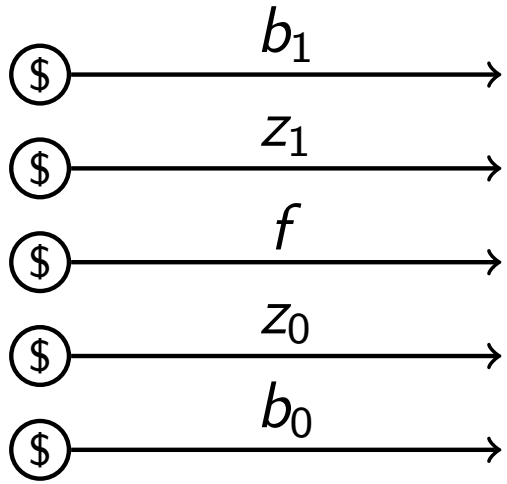
Using our PIOAs Hardness Assumption

Our composition and transitivity properties allow proving $SH2 \leq_{neg,pt} SHR2$:



$SH2$

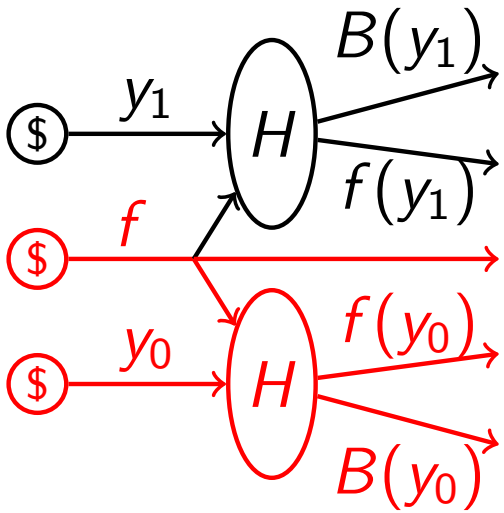
$\leq_{neg,pt}$



$SHR2$

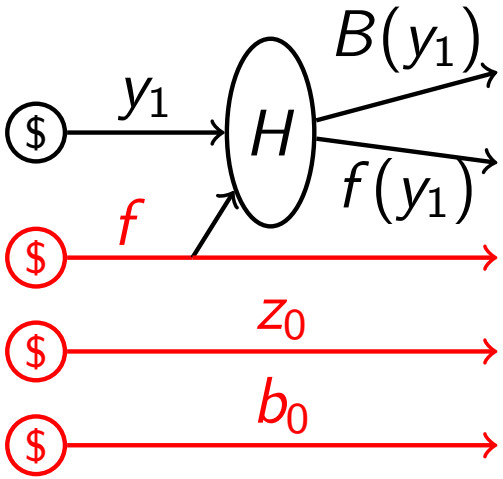
Using our PIOAs Hardness Assumption

Consider the *SHInt* intermediate system. We have:



SH2

$\leq_{neg,pt}$

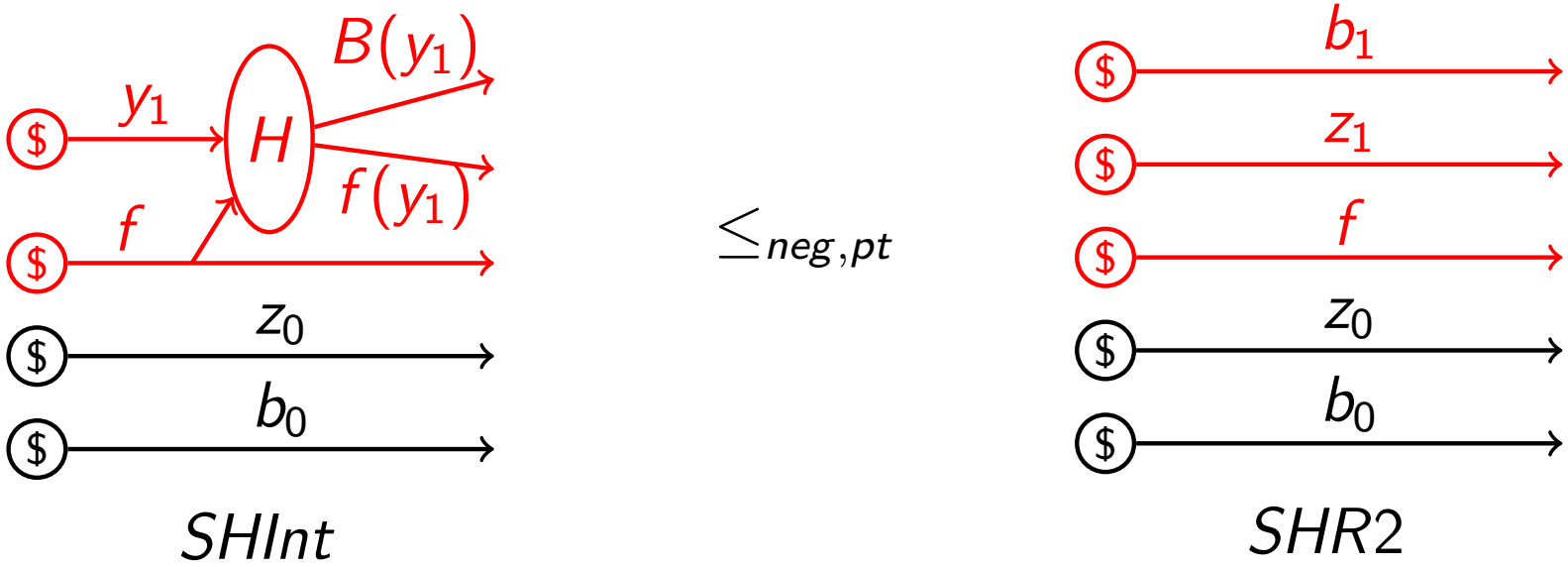


SHInt

SH2 and *SHInt* are just *SH* and *SHR* composed with the same systems!

Using our PIOAs Hardness Assumption

We also have:



SHInt $\leq_{neg,pt}$ *SHR2*

$SH2 \leq_{neg,pt} SHR2$ follows from our transitivity result!

Proving $T_1 \leq_0 T_2$

How do we prove that $T_1 \leq_0 T_2$?

or How do we prove that, for every environment E for T_1 and T_2, every trace distribution of $T_1||E$ is also a trace distribution of $T_2||E$?

$\Rightarrow$ We use a simulation relation!

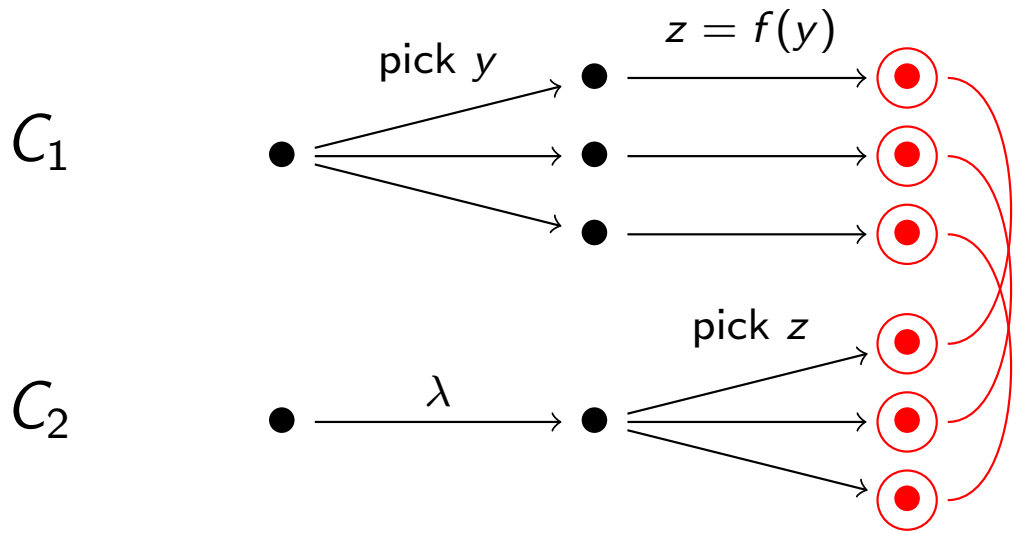
- Standard technique in the concurrency community, extended to our framework

Simulation Relation

How does our simulation relation R look?

- Suppose E is fixed. R relates ($\approx$):
  - distributions on states of $T_1||E$ to
  - distributions on states of $T_2||E$
- R is a simulation relation iff
  - start state of $T_1||E$ related to start state of $T_2||E$
  - for every task of $T_1||E$, there is a sequence of tasks for $T_2||E$ such that:
    - executing these tasks on both systems preserves traces
    - the resulting distributions on states are also related
- *Theorem:* If, $\forall E$, such an R exists, then $T_1 \leq_0 T_2$

Simulation Relation



R usually contains requirements like:

- if $C_2.zval = \perp$ then (1) or (2) holds:
  (1) $C_1.yval = \perp$
  (2) $C_1.yval$ is the uniform distribution on $Dom(f)$
- $C_2.zval = C_1.zval$

Conclusion

Case-study on a simple OT procotol [GMW87] available: MIT-CSAIL-TR-2006-019, Mar. 2006.

We hope task-PIOAs provide a framework for:

- More general, expressive, specifications
- More general, systematic, security proofs

Conclusion

Further works:

- General theorem for secure protocol composition
- Deal with other computational assumptions
- New case studies (key exchange, . . .)
- Mechanization
- New simulation relations
- . . .